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Investigating the product life cycle concept: an application to capital recovery; evaluation within the telephone industry

Manoj P. Dandekar
Iowa State University

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capital recovery; evaluation within the telephone industry**

Dandekar, Manoj P., Ph.D.

Iowa State University, 1987

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**Investigating the product life cycle concept:
An application to capital recovery;
evaluation within the telephone industry**

by

Manoj P. Dandekar

**A Dissertation Submitted to the
Graduate Faculty in Partial Fulfillment of the
Requirements for the Degree of
DOCTOR OF PHILOSOPHY**

**Department: Industrial Engineering
Major: Engineering Valuation**

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Signature was redacted for privacy.

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1987

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DEDICATION

Dedicated to

my parents

Malan and Phirozshah Dandekar

and

the late

Dr. G. E. Lamp, Jr.

I. INTRODUCTION

Contrary to the common belief "haste makes waste", depreciation practitioners, especially in the telephone industry in the 1980s, would rather make haste to recover capital invested in some categories of property than let it be wasted as unrecoverable capital in the rate base from where it will be finally removed as investment not in service. This attitude stems from the fact that the industry, and also generally the market, has had a very fast pace of technological developments recently. Coupled with competition, some from unforeseen quarters such as by-pass, the economic lives of many of its equipment investments have been dramatically reduced. This has caused considerable stress in the capital recovery process, rendering a huge reserve deficit, some of which may be nonrecoverable.

The philosophy for breaking even at a profitable level in manufacturing a product, or providing a service, is portrayed by the revenue requirement equation for the regulated industries:

$$RR = OE + T + D + ROR(RB)$$

where RR is the revenue requirement, OE is the operating expense less depreciation, T is the taxes, D is the depreciation, ROR is the rate of return on the rate base RB. As much as this philosophy is applicable in any industry, it is prominently used in the regulated industry due to the monopolistic status of the industry.

It should be noted that depreciation has been singled out as an individual item. This is because depreciation alone accounts for a large percentage of the revenue requirement and in some companies,

e.g., Illinois Bell Telephone Company, depreciation is the single largest cost. [Letter from T. L. Cox (V.P. Finance, IBT) to W. J. Tricarico (Secretary, FCC), Ref: 1984 rescription of depreciation rates for IBT, July 20, 1984.] The high depreciation expense arises from the industry's capital intensive nature.

Due to its role, a close watch is maintained by both the company as well as the regulating body on the estimation and accrual of the depreciation expense. As the capital costs have to be distributed over the service lives of the assets, proper estimation of service lives is of prime importance in depreciation allocation and, therefore, capital recovery. Life estimation, or the development of estimates of average life and mortality dispersion, is the second step for the computation of annual depreciation charges. The first step is life analysis, or the investigation of past experience [32].

Life analysis is a statistical study of the historical patterns of the retirements of assets, to discover whether any practical inferences with respect to life can be drawn. The conclusion from these studies may be reliable, based on certain conditions such as uniform and consistent relationship between retirements and age, that it is a homogeneous experience over the period of this relationship, and that no material changes occurred affecting the series of data [33]. In this step historical records are scrutinized for their accuracy, and appropriateness; and also proper statistical methods are chosen for the application.

Life estimation, the second step, evolves from the application of

knowledgeable judgement to the statistical observations from the earlier step. Information gathered by interviewing personnel from accounting, engineering, and operations provides depth to the adjustments necessary to the historical studies for estimating lives of existing plant.

The traditionally used models of life analysis are Iowa type curves, Gompertz-Makeham formulae, h-curves, simulated plant record, turnover methods, etc. Majority of these models derive service lives as a result of estimating the relationship of age to the forces of mortality. The forces of mortality are broadly categorized as:

- a) physical condition (wear and tear from use)
- b) functional inadequacy (obsolescence)
- c) unrelated causes (acts of god, management policy) [31].

Of these causes for retirement of property, physical condition as a force of mortality is comparatively easier to quantify. Moreover, the models have been developed based on data of assets majority of which did not "die a natural death", but were replaced because of obsolescence and inadequacy [32]. However, the other two being unpredictable by nature, cannot be inferred or deduced from purely statistical analysis of experience; they have to be subjectively conglomerated into the estimation of service lives.

The subjective inclusion of these forces of mortality makes them susceptible to counter arguments. Thus in establishing an estimate of service life, statistical analysis of historical data is given more weight than subjective input. Coupled with the commonly objected

assumption that plant in use or being installed resembles plant which has been or is being retired, the traditional methods of life analysis seem to portray a weak argument for estimating shorter lives especially when, in the recent past, technical progress and competition are judged to be dominant forces of mortality. This has been shown later wherein based on limited experience of certain early placements the life indication from pure application of the models is quite high. At that time, being impossible to visualize the technical progress and competition in the coming years, estimation of service lives of those placements was more in tune with the models leading to low depreciation rates.

A. Age vs. Time

It is of common knowledge today that technological progress has been very rapid recently, especially in the electronics industry. Computing power of older generation main frame computers is now available in hand held calculators; communications are highly enhanced not only with higher voice channel capacities but also the availability of video and data base transmissions. Technological evolution is fed not only by the technologist but also by the entrepreneurs and competition. A prospective competitor can offer a product differentiated (e.g., price, quality, etc.) from the existing due to lower market entry cost facilitated by technical progress. Simultaneously, competition drives technology as each firm strives to maintain market share and profitability. Inflation also causes

consumers to seek more economical and efficient means of performing their functions making competitive offerings of new technologies more attractive.

Technological advancements thereby result in the replacement of older technologies with more efficient technologies. Although the process of replacement may not be instantaneous, the replacement rate is greatly increased. Plant and equipment are being discarded even before they are "worn out" because of technical obsolescence. Though well-controlled industries have been able to pace the introduction of improved equipment so that the return of embedded capital is more or less complete, the advances in technology, and stiffer competition make it less and less possible to continue this practice.

The concept of obsolescence by technological substitution is giving a new dimension to the process of capital recovery. The need to examine and identify individual forces of mortality, and objectively combine them for better life estimates has been conceptualized by some researchers [18]. It has been suggested that obsolescence and technological progress as a force of mortality be singled out, and studied separately [50].

Obsolescence is brought about by the invention, development, and availability of improved products or processes having same general characteristics, or totally innovative ideas. Age of an item is no longer a dominant consequence, the item can become obsolete overnight. Time, or rather the introduction of a competitive product in chronological time, is the decisive factor in the obsolescence of a

given product. Thus functional inadequacy of equipment in a particular operating environment is more due to time than due to its age.

Although it is difficult to represent absolute obsolescence of a particular item, time related procedures such as technological substitution/adoption can portray quite well the relative decrease in the popularity and use of that item. The decrease in use, or the reduction of the number of units in service for a product reflects its relative obsolescence with respect to certain economic and geographic environment.

Therefore, for certain types of property, traditional life analysis techniques which are developed on the basis of age, may need to be augmented with some time dependent procedures. In such cases of technically dependent property, time related procedures can provide additional information to give some objectivity for forecasting the future life characteristics.

It should be noted that whereas these time based functions can be used to present overall characteristics of service lives with respect to time, they lack the woven detail of traditional mortality analysis. The additional information provided by these time related procedures can give credibility to the "professional judgement" part of life estimation.

It is difficult to develop realistic forecasts of service lives using only age-dependent mortality analysis on property in which a good portion of the retirements are time related. Although methods of applying forces of mortality with weighting factors have been suggested

[18], the task of quantifying these forces is formidable. Time oriented models provide complementary and supportive information for the life estimation of property placed in the scenario of rapid technical advances.

B. A Case Study

Some of the most rapid technological advancements have occurred in the telecommunications industry in the recent past. Switching systems have evolved from manual switching through electro-mechanical to electronic systems. Even within the state-of-the-art electronic switching systems, yesterday's new analog systems are being replaced by digital systems. Another example is of old copper wire used for transmission of signals being displaced by the more versatile glass optic fiber.

Although these changes did not occur overnight, timely depreciation accrual adjustments were not made to fully recapture the investment because of the unexpected high rates of retirement. Coupled with reorganization and expanding competition, many telephone companies are faced with large depreciation reserve deficits created by these rapidly changing service lives. Just before divestiture, AT&T reported its estimated depreciation reserve deficit to be over \$25 billion and growing at a rate of \$2 billion a year [21b].

To maintain the competitive edge, some companies may have to forfeit some of this deficit. Their position would be like firms in post-industrial revolution Britain. Learning from the pioneering

efforts, other countries started with better machines while Britain had to compete using the obsolete prototypes. The steel industry has also faced a similar scenario where large amount of capital is yet to be recovered from outdated technology.

Among other reasons, some practitioners have held traditional life estimating techniques responsible for the reserve deficiency. This may be true based on the earlier discussion, where in spite of the versatility of the existing techniques shorter lives reflecting the future higher retirements due to technology and competition could not be convincingly estimated. Giving the benefit of doubt, it would be advisable to accept the augmentation of time related procedures to the traditional methods of life estimation. The time related procedures provide an umbrella beyond which the investments in a particular account may not survive, an envelop within which the traditional models should be applied.

Of the several time related procedures, the Product Life Cycle (PLC) concept is the contender for discussion herein. Following the objectives of this study, the next chapter presents the PLC concept along with a discussion of the literature cited. Though not exhaustive, the review of literature is designed to give the reader an overview of the PLC concept, its origin, and its applications.

Chapter IV consists of a discussion of the relation of mortality analysis over the life cycle of an account. Observations and inferences from the analysis of real data are presented in this chapter.

In the fifth chapter a life cycle model is presented. The model is a "standardized" one in that it has been derived without reference to any specific technical context or time frame. Some underlying assumptions made in the development of the model are mentioned. The goodness of fit of the model is checked against some real data.

The sixth chapter deals with the development of type life cycle curves based on the generalized model. Generalized type curves have been identified, and a method of application is demonstrated.

In chapter seven the investment life cycle is applied to the process of capital recovery. Remaining life of the embedded plant can be found from the projected recovery life and the weighted average realized life. This leads to the derivation of the required accumulated depreciation of the present plant. The final chapter briefly summarizes the material presented and provides conceptual insights and suggestions for further developments.

II. OBJECTIVES OF STUDY

It was not until 1980 that the FCC through FCC docket 20188 [17] authorized the use of equal life group procedure, and allowed the application of the remaining life methods. Even though these concepts have been around since early forties [31,48], it took quite some time to "legalize" them. One can very well imagine the time it will take for the product life cycle concept to be accepted as a means for capital recovery.

The subject of this study is the application of the product life cycle concept to the process of capital recovery. The phobia of something new may be reduced by understanding how it is related to something known and understood. It is therefore necessary to explore the life cycle concept and try to fit known characteristics of life analysis within its framework. In this process a workable procedure marrying the age dependent mortality analysis and time related life cycle can evolve.

The specific objectives of this study are:

1. To investigate the product life cycle concept and to find out how known characteristics of life analysis such as average life and mortality dispersion can be related to this entity.
2. To develop a mathematical model to represent investment life cycles and check its validity against real data.
3. To develop generalized type curves based on this model and if possible, present a working procedure to estimate capital

recovery life using the life cycle type curves.

4. To develop a methodology to find the remaining life and the required accumulated depreciation of embedded plant based on projected investment life cycle.

III. LITERATURE REVIEW

The idea of product life cycle can be thought to have stemmed from biological sciences. The basic observation that products like multicellular organism pass through definable stages of birth-growth-maturity-death over the span of their position in the market, led to the introduction of the Product Life Cycle (PLC) concept.

Although much has been said and written about life cycles in the marketing management literature since early fifties its application in the capital recovery area was suggested as recently as the early eighties. Based on applications, literature on product life cycles can be broadly categorized into product life cycle concepts for marketing, and product life cycle concepts for capital recovery. Majority of the contributions are in the areas of strategy management planning, and mathematical modelling of product diffusion to the consumers or market.

A. General

Little does a beautiful butterfly know that it has emerged only after it had passed through a series of ordered events, that is, an egg to moving around being a larva and undergoing metamorphosis in a cocoon. It then goes about its business when, finally, one day it passes away. Although the time span for each stage is not as closely defined as in the case of a butterfly for an individual, mankind also experiences a fixed sequence of the stages as one passes through conception, birth, infancy, adolescence, maturity, old age, and finally death. It has been observed that such stages, as experienced by living

organisms, are also exhibited by inanimate products, systems, and processes.

In likening the stages for mankind to a theoretical cycle for a product Wasson [47] has summed up the following (Fig. 1):

- 1) Conception of a performance package capable of fulfilling related set of consumer desires.
- 2) An incubation period of product development.
- 3) Introduction of product and market development.
- 4) A period of rapid growth.
- 5) Maturization - competitive turbulence.
- 6) Stability and saturation.
- 7) Decline and substitution.
- 8) Death and replacement.

Of the above eight stages only the last six stages represent the product during its public life, that is, when it is in the market. Most researchers consider the market phase of a product for life cycle analysis and adopt a four-stage cycle - introduction, growth, maturity, and decline (Fig. 2) The characteristics of the stages are as follows:

- | | |
|---------------|---|
| Introduction: | Slow start, small production capacity, technical problems, customer reluctance. |
| Growth: | Customer acceptance, large demand, increased competition. |
| Maturity: | Substitution initiated, intense competition. |
| Decline: | Sales drop, superior alternatives available. |

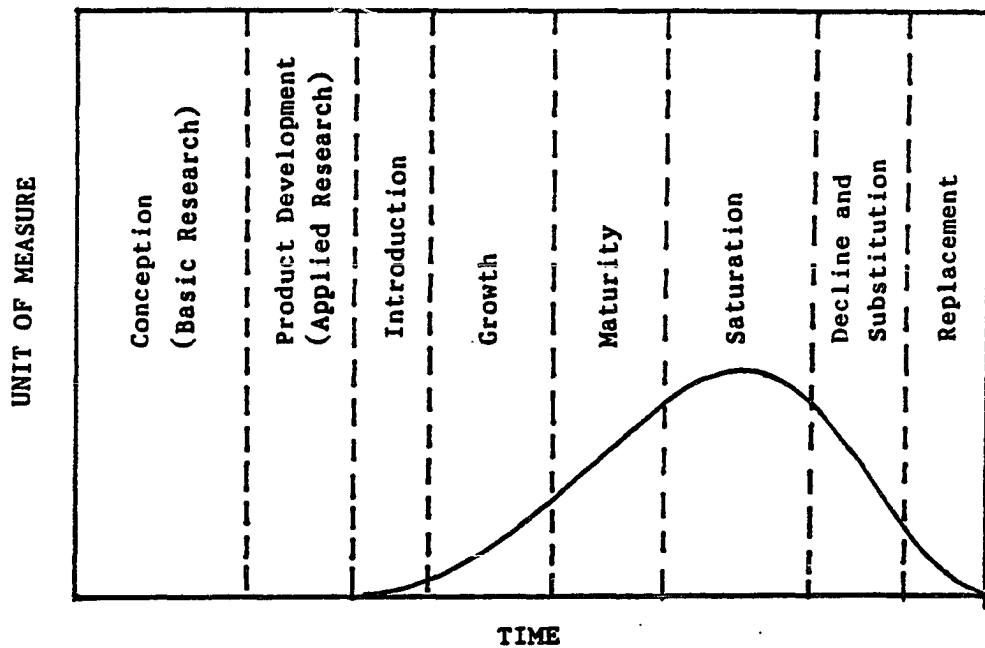


FIGURE 1. The full product life cycle

The definition of life cycle can be generalized to be the status of an object (living or inanimate) at different points in time from its inception to expiration as it passes through identifiable stages. The status of the object can be defined with respect to some unit of measure (or measure quantity) such as annual sales volume, net profit, units in service, etc. Although a typical life cycle curve is represented graphically by a gradual build-up, leading to a mature peak, followed by decline, the configuration largely depends on the units defining the status of any given object. Thus the shape of the life cycle for the annual sales of a product is different from that for

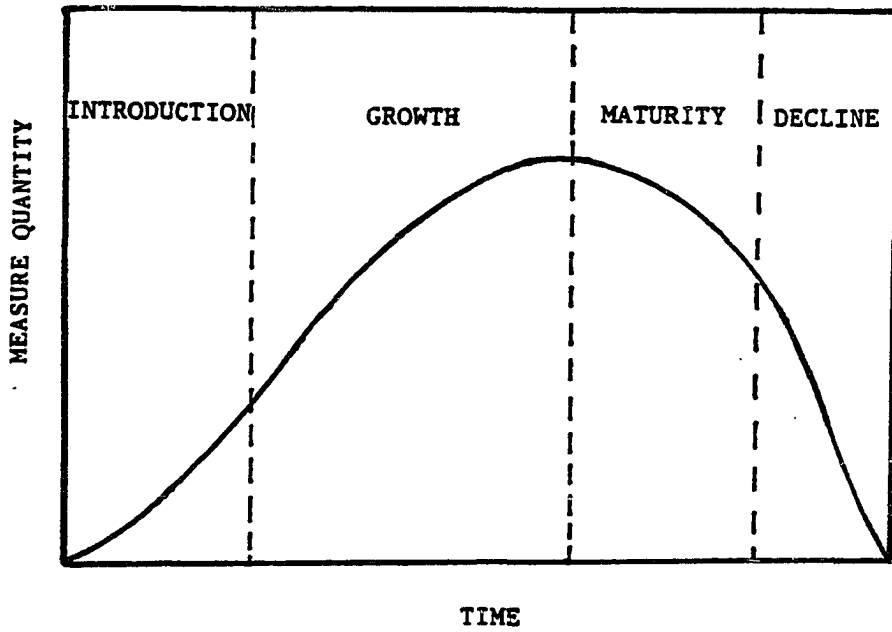


FIGURE 2. Four stages of a product life cycle

the units in service or invested of the same product (Fig. 3)

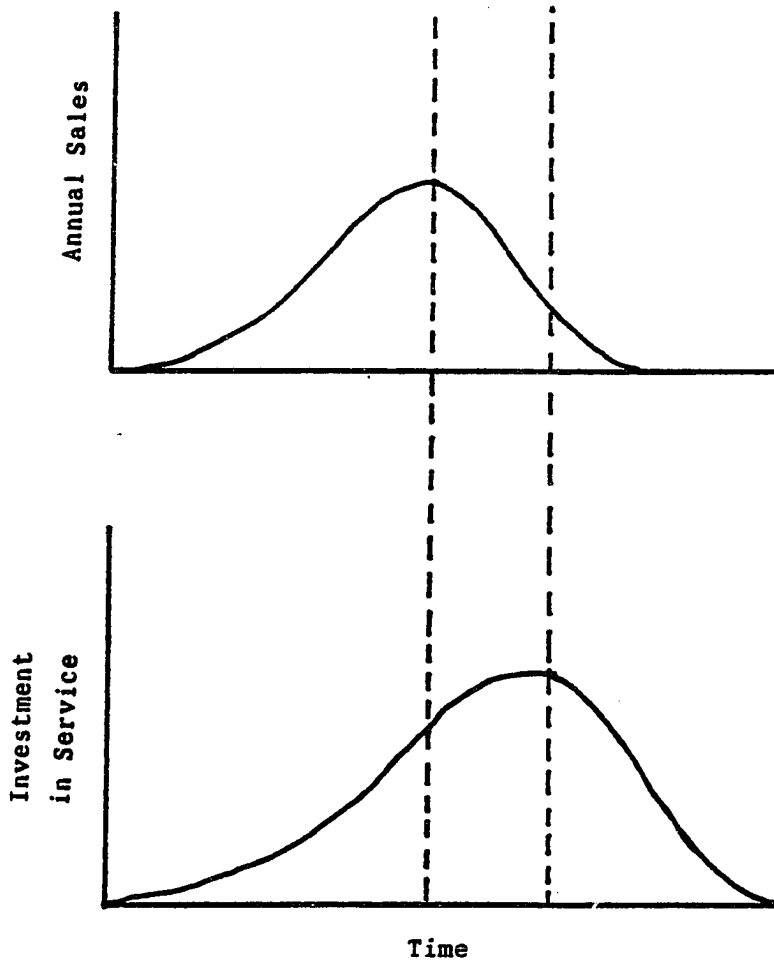


FIGURE 3. PLC shapes for different measure quantities

The definition of the product is as important as the unit of measure of the object considered. Polli and Cook [36] find it meaningful to distinguish between product classes, product forms, and product brands. The three levels of product aggregates are defined as:

Product class: include all those objects
that despite differences in shapes, sizes,
and technical characteristics are essentially
substitutes for the same needs.

e.g., cigarettes, cigars, and pipes;
 mechanical, electro-mechanical,
 and electronic switching systems.

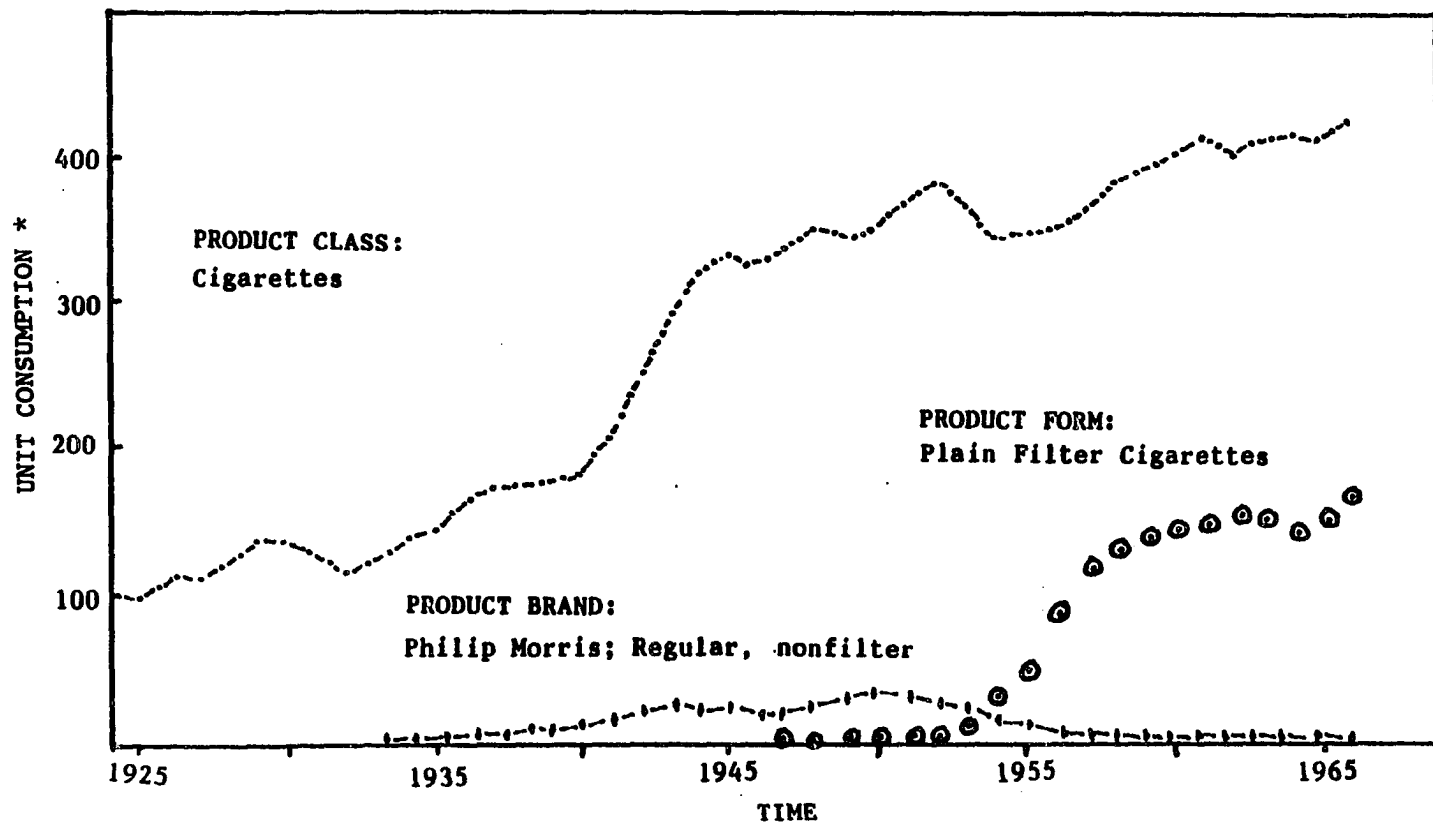
Product forms: include all those objects that,
 though not identical, are technically
 quite homogeneous.
 e.g., king-sized, regular,
 and menthol cigarettes;
 panel, step-by-step,
 and crossbar switching systems.

Product brand: include those objects that are
 completely specified technically,
 and are further identified trade marks, etc.
 e.g., Camels, Kool, etc., cigarettes;
 Western Electric,
 Northern Telecom switching systems.

For a given unit of measure the life cycles for the different
 aggregates for cigarettes is shown in the Fig. 4. It has been observed
 that product classes possess the longest life cycles, and while product
 forms demonstrate more typical life cycle configurations product brands
 usually exhibit erratic trends.

Furthermore, for any given unit of measure, each product has its
 unique life cycle. The shape of the life cycle also depends upon,
 among other factors, obsolescence of existing products [37], time and
 sales of a closely related product [6], changing market and economic
 conditions [15], managerial and government intervention [43]. Although
 ten different life cycle (Fig. 5) patterns have been identified over
 the years by researchers [38], it could as well be that some of these
 are a part of an overall general life cycle whose time spans are in
 decades or even centuries.

The confusion arising from all this has led some researchers to
 advise managers to use efficient information systems rather than to



* Number of cigarettes per 100 of constant dollar nondurable consumption.

FIGURE 4. PLC shapes for different aggregates [36]

rely on the life cycle notion [14]. Abell [1] defines a product as "the application of a distinct technology to the provision of a particular function for a specific consumer group." As product forms, the above definition so closely defines, bear the most likely approximation to the traditional life cycle, it is widely used in research.

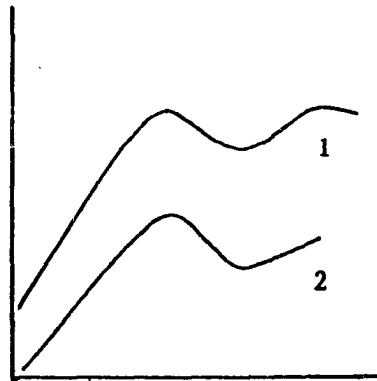
In marketing parlance the product life cycle represents the pattern of projected or historical sales either in dollars or in units for a product extending from the time it is first placed in the market until it is removed [38].

B. Management Strategies

The main application suggested for the life cycle has been to plan marketing strategies as the product moves from stage to stage. Coupled with a "product-process matrix" [24], the product life cycle concept is a very versatile tool for corporate strategy planning and financial administration [21a]. Hofer [25] has even proposed that "the most fundamental variable in determining an appropriate strategy is the stage of the product life cycle."

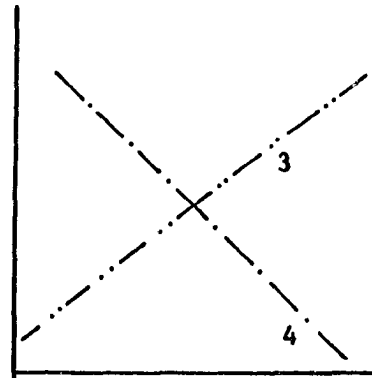
Although the relationship between market capacity and market volume is a good indication of the stage, complementary analysis of the rate of change of the sales volume and the profit/loss curve provides a better measurement of the life cycle stages [41].

Polli and Cook [36] suggest the use of percent change in a product's real sales from year t to year $t+1$. Assuming the



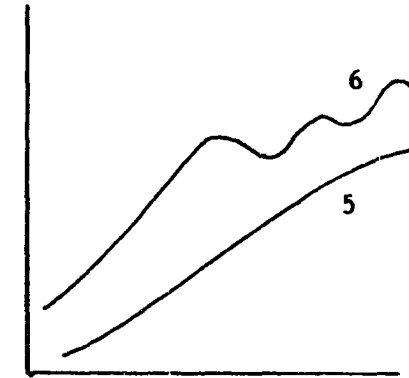
1. Cycle-Recycle

2. Cycle-Half Recycle



3. Increasing Sales

4. Decreasing Sales

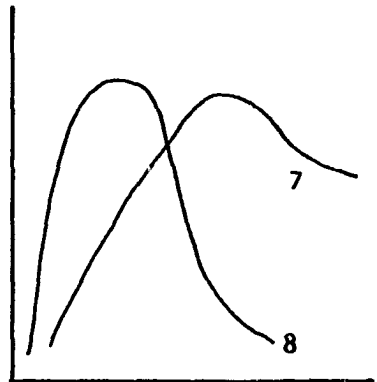


5. Growth Maturity

6. Innovative Maturity

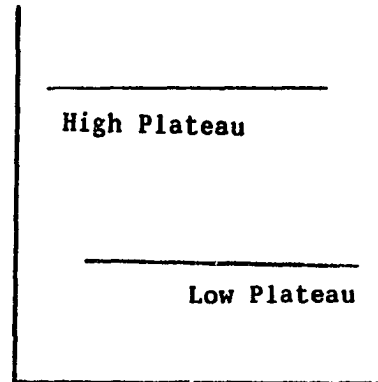
X Axis: TIME

Y Axis: SALES REVENUES

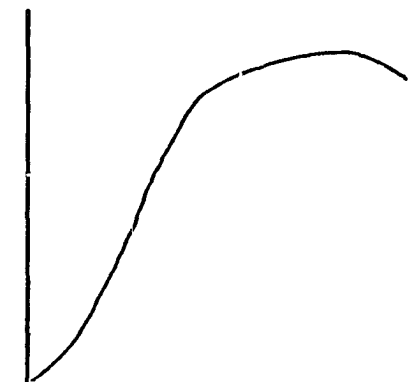


7. Growth-Decline-Plateau

8. Rapid Penetration



9. High and Low Plateau



10. Stable Maturity

FIGURE 5. Different patterns of PLC

distribution of these changes to be normal, boundaries can be assigned to identify the stages of the life cycle, as follows: values lesser than $-1/2 \sigma$ can be considered to represent the "decline" stage; values greater than $+1/2 \sigma$ represent "growth"; and values in the range of $\pm 1/2 \sigma$ can be considered to be stable or represent the "maturity" stage (Fig. 6). Though a mathematical basis has been suggested, most researchers and practitioners use subjective demarcations such as changes in sales volume, and other trends to identify the different life cycle stages.

Introduction stage: Here sales volume is low but growing slowly. This is due to the inertia of market resistance of accepting a new idea. The introductory stage is usually characterized by a loss due to the high initial outlay for product and market development. Both sides, seller as well as buyer, are in the learning phase. Research and engineering personnel should locate and remedy defects if any. The objective here should be to minimize learning; develop awareness of the benefits, and present the product as a distinctive superior serious alternative.

Growth stage: Having survived the introductory phase, product acceptance is marked by increased sales having increasing growth rates. Capital intensity increases as mass production methods are gradually introduced. Manufacturing becomes the key function as research and engineering have played with the design long enough for competitors to try pre-empting the market. Management should try to establish a strong brand market and set up an efficient distribution systems.

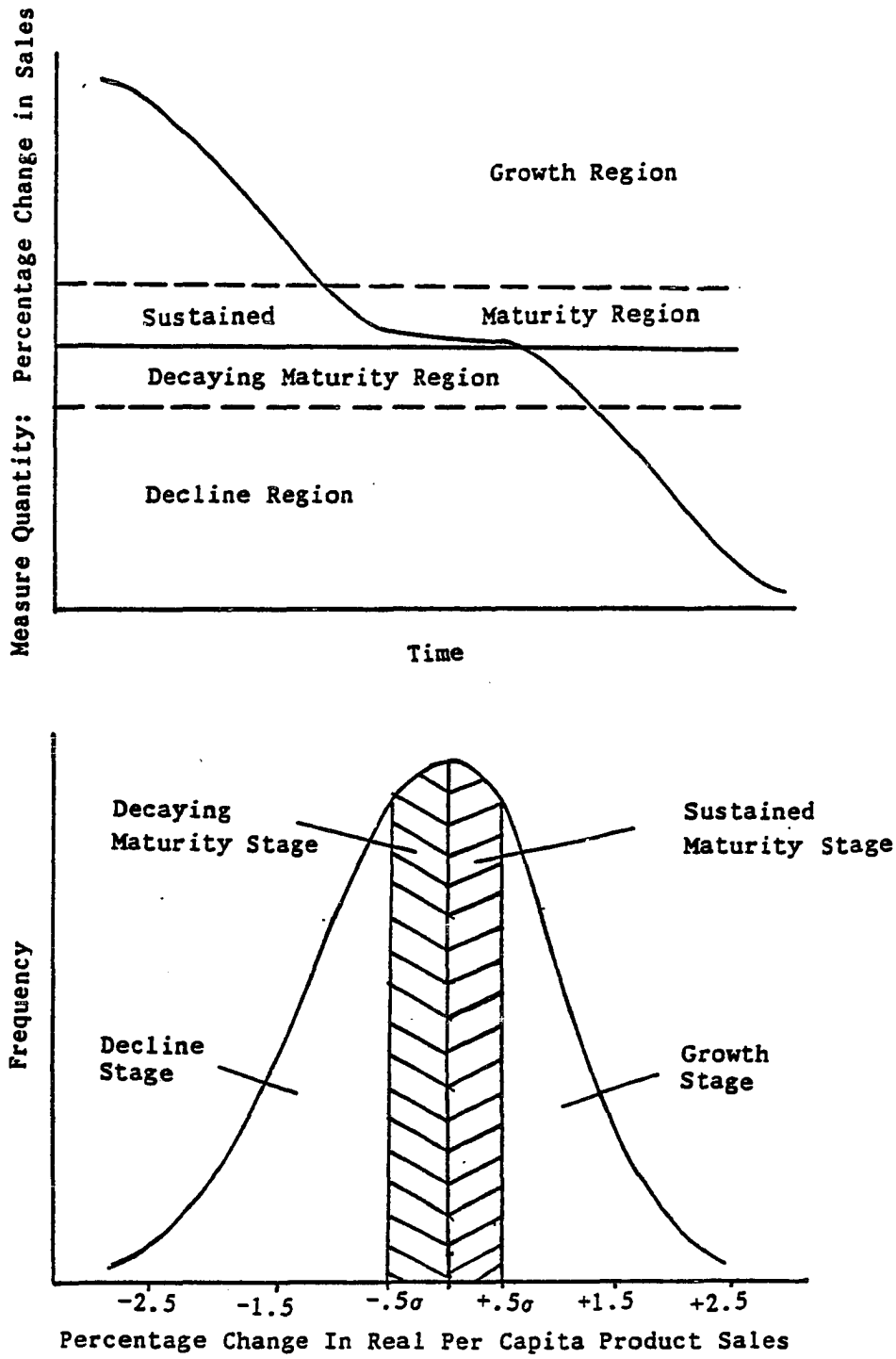


FIGURE 6. Stage identification based on percentage change

Instead of seeking ways of getting the customer to try the product, the management faces a more compelling task of getting them to prefer the brand [27]. This is a period of high and rising profits for manufacturer, distributor, and retailer.

Maturity stage: As the volume rises, the market becomes increasingly saturated. At this point even the competitive products become reliable. Improvements in the product tend to be small, and there is less to choose between competitors that product standardization sets in. It is necessary to hold brand preference by finer product differentiation, customer services, and promotional practices. To remain in the competition an updated product can be reintroduced [42]. Despite rising volume, needing more unskilled and semi-skilled labor, the profit margin begins to slip. Creative selling and pricing policies will carry the product through this stage, and may even open new venues for it. Although profits are shrinking, they are still enough to attract new competitors.

For appropriate pricing in this stage, the three aspects of maturity, developing almost simultaneously, should be considered:

- (a) technical maturity, indicated by decreasing rate of product improvement, standardization among brands, and stabilization of manufacturing processes;
- (b) market maturity, indicated by consumer acceptance, comparison of competitive brands by well informed customers with confidence that they perform equally satisfactory; and
- (c) competitive maturity, indicated by the stability of market shares and price structures [13].

Decline stage: As the product matures the cycle is characterized by a movement from labor-intensity to a relative capital-intensity [46]. The increased product maturity calls to reduce cost in each step of the engineering-manufacturing-distribution systems. Sales volume tends to decrease due to sophistication of customers to evaluate price and quality more effectively. The attractiveness of the new products and substitutes play an important role in the consumers' assessment of the product. Unless enthusiasm in the product is revived, the rate of change in sales will reduce to a point where costs may surpass returns. The strategy here is one of retreat; phase out the old product, and seek new markets leading to new horizons.

Although no unique strategy exists for each stage, some actions are more effective than others depending on the stage of the life cycle [46]. Using trajectory and timing the product life cycle can be used as a framework for developing plans, and measuring performance [44].

C. Diffusion and Adoption

Although the value of the product life cycle model has been emphasized as a basis for product planning and control [40], it is presented mostly as a qualitative concept. Many writers do not try to quantify the product life cycle while endorsing its use as a framework for management analysis.

The characteristic life cycle curve is more or less bell shaped. Lack of awareness cause the sales to be low in the introduction stage. The growth stage initiated by consumer acceptance has sales increase at

an increasing rate. However, with the entry of competitors the growth rate is checked; and with the market becoming saturated the sales reach a plateau in the maturity stage. As consumers seek newer substitutes, for whatever reason, the sales decline when finally removal becomes imminent.

The basic curve form describing the above scenario may be represented by a simple parabola with the equation:

$$Y = a + bX + cX^2$$

where Y is the measure quantity and X is the time axis. In his study of product life cycles for ethical drugs Cox [11] found a high percent of the products demonstrated a life cycle which could be represented by a fourth order polynomial. It has been suggested that the shape of such a life cycle curve improvised on the basis of Pearsonian distribution can be used as a predictive model [2].

Assuming that a life cycle consists of three parts: growth of sales in the initial period; decline in later stages; and an overlapping period influenced by the sales of comparable products, Brockhoff [6] proposes the following model:

$$Y = a + bX^2 + c/X + dR$$

where a, b, c, d are parameters and R is the sales of comparable products. The function being rather inconvenient, he suggests the use of a nonlinear form

$$Y = aX^b \exp(-cX) + dR$$

which is essentially a Gamma distribution.

The rationale for product life cycle concept finds its roots in

the theory of diffusion and adoption of innovations [19] as can be seen from the shape of the characteristic life cycle curve. The most noted empirical regularity of diffusion over time is that the graph approximates an S-form [7]. The S-shaped graph can be produced by a number of functions under different sets of assumptions describing the process. Although it is not the most plausible function [7,36], the logistic is the most often used function for generating such curves. This could be due to the ease of estimating its parameters, and that the parameters, treated as descriptive measures of the diffusion process, can be used as dependent variables for further analysis [22].

A modified version of the logistic by Bass [3] is a more accepted model for new products. Arguing that the traditional new product diffusion models have assumed a constant market potential, Mahajan et al. [29] have presented a dynamic model which relaxes this assumption. Various new product models have been developed to portray the growth and maturity phases of a new item but they are unable to depict the second half of the product life cycle.

Assuming the life cycle model to be a continuous real valued function of continuous time, Cooke and Edmondson [10] suggest the use of differential calculus to determine the boundaries of the various phases. They have developed a computer program that uses early data points to estimate the remainder of the life cycle.

As against the common notion for most models of new products, Harrell and Taylor [23] include replacement purchases in the life cycle volume. This product life cycle shown in Fig. 7 ignores the

substitution that may subsequently take place. In addition to repeat buying, intervals between purchases make significant improvement in the life cycle model [20].

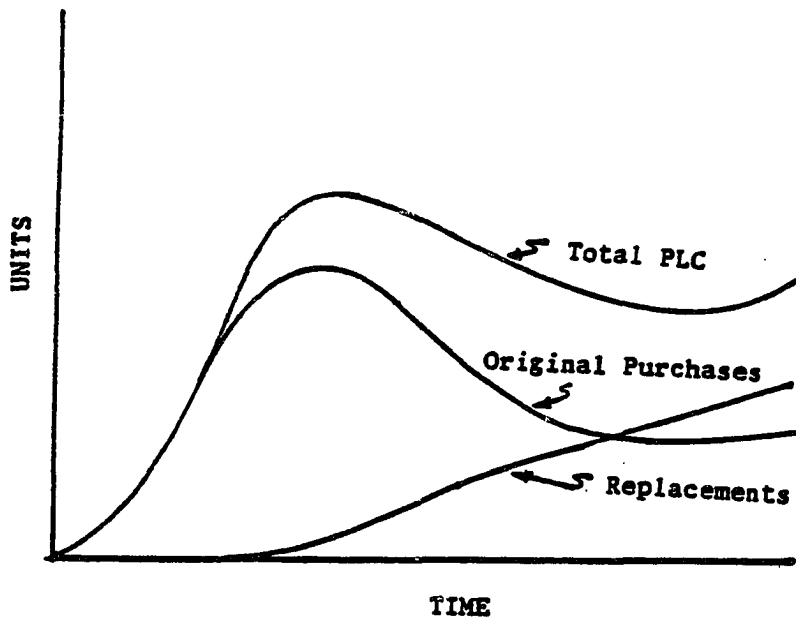


FIGURE 7. PLC with replacements

D. Life Cycles in Capital Recovery

The closest that a nontechnical researcher comes to capital recovery is Neidell [34] where he differentiates between a profit curve and an investment recovery curve (Fig. 8). His definition of investment recovery which is equal to sales revenues less expenses is very much attuned to that of profit; and it is far from capital

recovery as defined herein. The phrase "capital recovery" infers that proper adjustments in the financial statements are made to recognize the consumption of the capital over time.

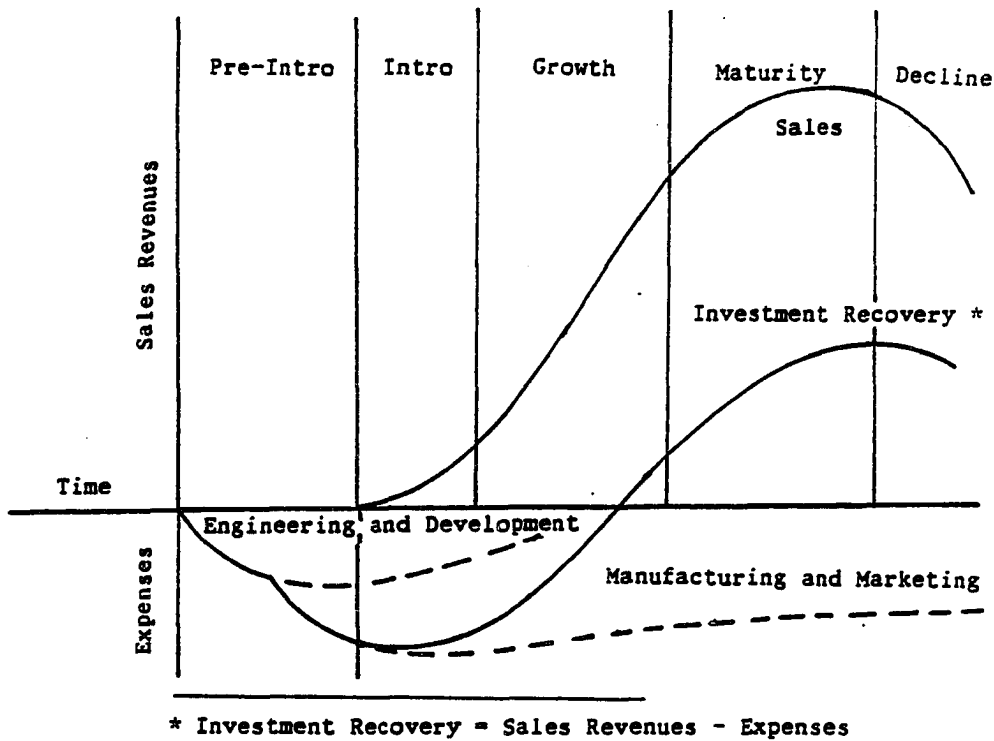


FIGURE 8. Investment recovery along PLC

The idea of using life cycle concept for capital recovery was introduced to the Bell System in 1979. A delphi approach has been suggested wherein capital recovery personnel work with subject matter experts (SMEs). The SMEs first disaggregate the investment into groupings of like products or functions; the groupings should be able to readily lend themselves to analysis. Next, principal life cycle

stages are identified for each grouping along a time line.

By placing the different time lines along a common time scale, the evolution of successive generations as well as relation between the generations of the groupings can be visualized. This also helps to confirm the approximate dates from the earlier step. Using this and technical knowledge of competition, technology, and economy life cycles can be forecasted to completion.

Based on this general scenario Ocker [35], and Clark [9] suggest that the life cycle of a future grouping may follow that of a present grouping. However, they disagree on the choice of the present grouping.

As the recovery of capital becomes more critical during the decline stage of the life cycle, it is suggested that this stage be further divided into three phases of selective retirements, moderate retirements, and rapid retirements. This presents a better picture to the capital recovery analysts for their study. Johnson [26] suggests the use of this information for calculating remaining life on the assumption that the future retirements are independent of age.

IV. MORTALITY ANALYSIS AND INVESTMENT LIFE CYCLE

The status of a product describes its life cycle at different points in time as it passes through identifiable stages over its life span. An unit of measure or life cycle measure quantity, be it by annual sales, or units in service, or annual investment, etc., defines the status of the product. For capital recovery and financial management purposes units in service, and annual investments and balances are more appropriate than annual sales and allied units (which are useful in marketing management) to be chosen as life cycle measure quantities.

A. Balances as Measure Quantity for Life Cycle

Uniform System of Accounts requires that property be maintained to reflect physical quantities and dollar original costs. Accounts are standardized for different industries by grouping property comprising of units having particular general character or use; each account is identified by a number. Examples in the telephone industry for telephone plant accounts are: land (211); buildings (212); central office equipment (221); station apparatus (231); pole lines (241). Some accounts may be further subdivided into subaccounts of property units very much similar. Central office equipment (C. O. E. 221) account is further divided into manual (17C); step-by-step (37C); crossbar (47C); radio (67C); electronic (77C).

Information systems can convert data available from the accounting department into formats for finance management purposes, and those

prescribed by regulatory bodies. These data of additions, retirements, adjustments, and balances are reported on monthly, quarterly, and annual basis. The information is usually in dollar amounts but can also be compiled in units in service. As balances are the resultant of the additions, retirements, and adjustments at the end of a reporting period, they are deemed to be the representative measure quantities for life cycle analysis purposes; be they in dollars or units in service. The balances represent the resultant investment in a particular property account.

Among these four entities: additions, retirements, adjustments, and balances, additions come close in the choice for the measure quantity describing the life cycle. While retirements occur at the end of a service rendered, adjustments may be termed sporadic depending on acquisitions, sales, or accounting reclassifications. Additions very much comply with the shape characteristics of a product life cycle: When the product or process is new, the management invests in it on an experimental basis. As it attains confidence, the bugs are removed, more investments are made. Annual additions represent replacement as well as expansion in the industry; this is in tune with the growth and maturity stages of a product life cycle. Demand saturation or the introduction of a new technique may then cause the management to revise its policy of investing in the existing technology and the declining stage sets in with no more additions taking place. Although nominal additions may be made to continue the serviceability, no major appropriations are encouraged.

As much as additions follow the characteristics of a product life cycle, they cannot be used as the units of the life cycle concept for life analysis purposes. Additions, annual appropriations and capital investment as they are, do not represent the service rendered from them. Service rendered is represented by the amount of the additions in a year continue to be in operational use in the following years until all those installations from that year are totally retired. Balances, usually in terms of end-of-the-year balances, on the other hand represent the survivors of the preceding years' additions; and therefore the service rendered till finally removed from operational service.

B. Relating Life Cycle to Turnover Method

End of the year balances are computed by adding investment made in that year to the balance at the end of the prior year less the retirements during the year; adjustments, that is transfers in or out of the account or account reclassifications, if any are also to be included. They are also calculated by summing up the survivors of all the vintages placed into service so far. Mathematically, end-of-the-year balance

$$B_i = B_{i-1} + A_i - R_i +/- AD_i \quad (1)$$

where A_i are the additions in the year i , R_i are the retirements during the year i , and AD_i are the adjustments made in the year i for the former computation method. In the latter case

$$B_i = \sum S_i \quad (2)$$

where S_i are the survivors of all the preceding years investments.

Neglecting adjustments and doing progressive substitution Eq. (1) can be converted to

$$B_z = \Sigma A - \Sigma R$$

where ΣA is summation of all additions and ΣR is the summation of all retirements upto year z from the first year in the account under consideration. At this point it should be pointed out that the terms of the right hand side can be used for calculating an estimate of average life of property. The method that uses these accumulated amounts is the turnover method.

In the turnover method, it is assumed that the average life is equal to the number of years that it would take to accumulate retirements equal in number to those units in service at any given date; it is the period of turnover [49]. When the accumulations of the additions and retirements are plotted according to the years, the horizontal distance between the two curves gives the turnover period which is equivalent to the average life (Fig. 9). Thus, whereas the horizontal distance reflects the average life, the vertical distance between the two curves represents the amount in service.

The use of turnover method is restricted to property in continuous service, and where the replacements have similar potential life expectancy as the retirements. By the virtue of these conditions, the curves of the accumulated additions and retirements tend to follow a more or less parallel path. However, in the case of plant experiencing technological obsolescence, the accumulated additions curve stabilizes

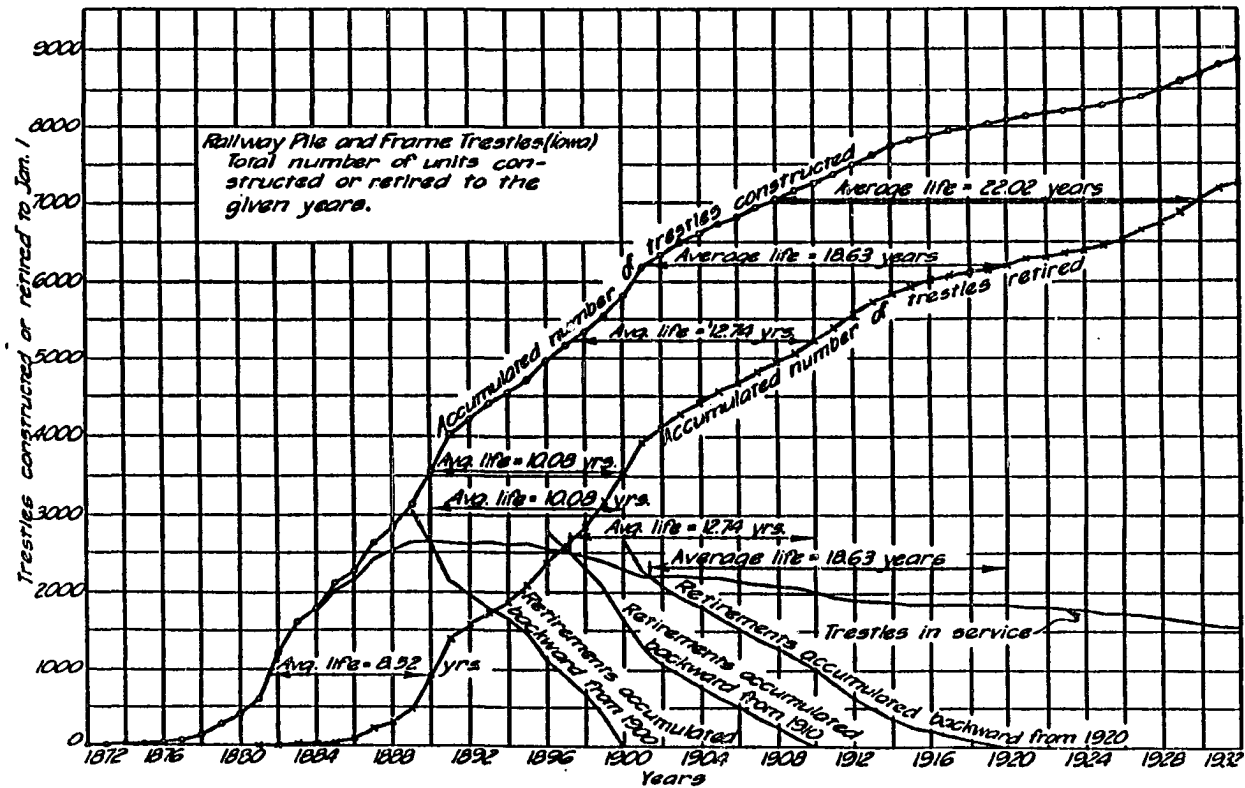


FIGURE 9. Determination of average life by the turnover method

as explained earlier, and the accumulated retirements slowly add up to the total investments when the account is finally closed. In this case the two curves converge to the point of maximum life cycle and the total investments in that account (Fig. 10).

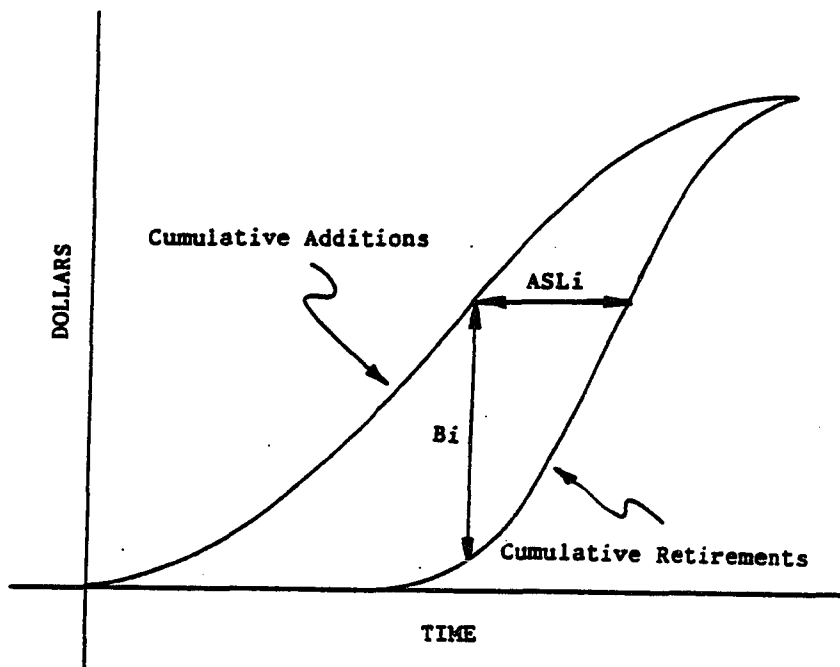


FIGURE 10. Turnover method approach over PLC

The area between the two curves then represents the total service obtained from the account. The vertical distance between them gives the balance or investment in service at different points in time. The balances start at a low amount, slowly increase to a maximum, and then decrease as obsolescence causes less additions to be made and more

retirements to be experienced. The horizontal distance between the two curves gives the average life at various points in time. The distance is quite large in the beginning, but slowly decreases as the two curves start converging. The above discussion is presented in Fig. 11 for actual data.

This data set was used by Clark [9] to demonstrate how the low depreciation rates induced by the earlier longer lives had caused a deficit in the company's depreciation reserve. Although he did not comment on the short lives of the latter vintages, Clark proposed the need for an overall shorter life to allow for the complete recovery of capital without need for amortization.

When the balances are represented by the summation of the survivors as in Eq. (2), the thought of disaggregating the summations to orient the survivors to their original placement immediately follows. Survivor curves of the original vintages can be observed as a result of this orientation. The area under a survivor curve represents the service rendered by that placement, while the area under the life cycle curve represented by balances gives the total service obtained from that account. It can be deduced that the area under the life cycle curve is the sum total of the areas under the individual survivor curves of the different vintages (Fig. 12).

The area under a survivor curve is the product of the original additions in a vintage and the average service life (ASL). Thus,

$$\text{area under a life cycle curve} = \sum (A_i)(ASL_i)$$

where A_i additions in vintage i have an ASL_i . Investment recovery life

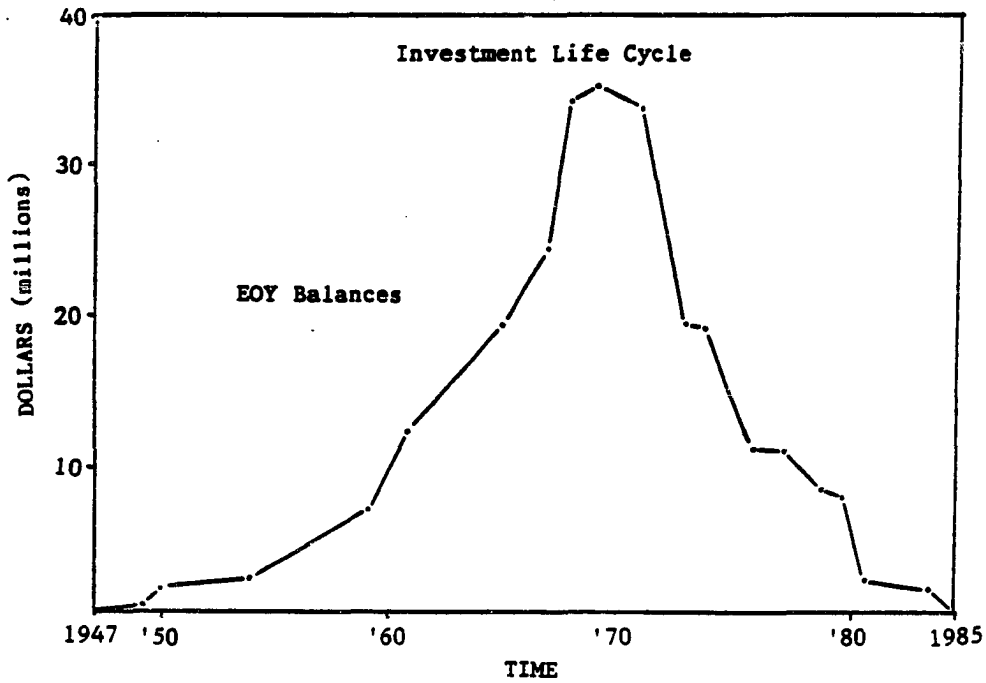
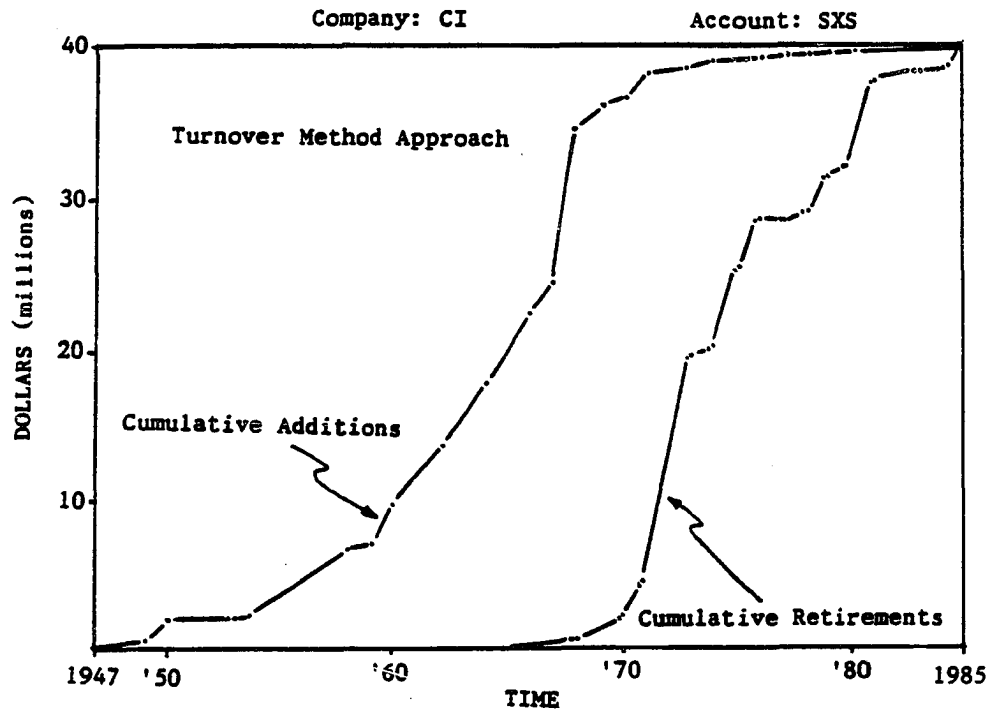


FIGURE 11. Turnover method approach and PLC

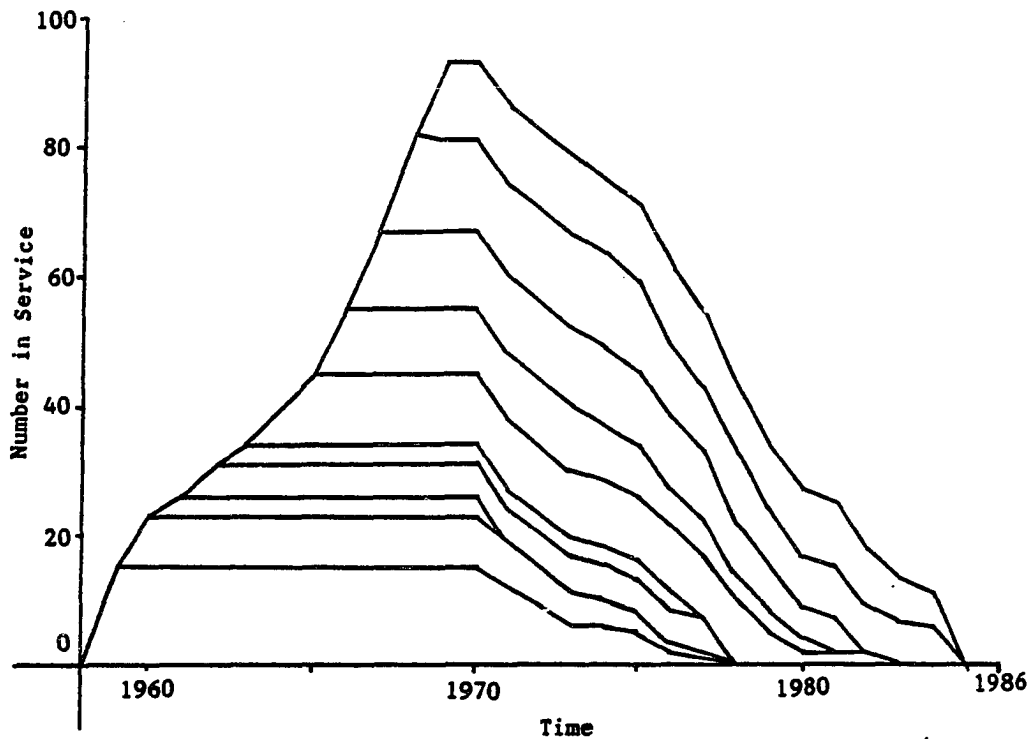


FIGURE 12. Survivor curves over the life cycle

(IRL) is the area under a life cycle curve divided by total additions in that account. Therefore, from the above equation, IRL is the weighted average service life of all the vintages in an account over its life cycle.

C. Mortality Analyses over Life Cycles

The shortening of service lives analyzed from the turnover method can also be demonstrated by actuarial analysis of accounts over their life spans.

1. Analyzing at the most recent time available in the subaccount, one can observe that the service life indicators

for the earlier vintages are much higher than for the recent placements; there is a declining trend in the service life indicators as one move from the old to the new vintages (Table 1).

2. The dispersion patterns indicated in the analyses shift from right modal for the older placements to left modal for newer property (Table 1).
3. Following any earlier vintage through its life span, one can observe the service life indicators decrease as the experience increases (Fig. 13).

These observations were generalized from the results of actuarial analysis performed on different types of data. Some of the results are presented in the appendix. A computer program (IBFIT) available at Iowa State University through the Department of Industrial Engineering was used for the actuarial analyses of the accounts. Mortality analysis performed on individual vintages placed into the accounts provided information regarding their average lives and dispersion patterns. The computer runs were made for two scenarios. In the first case using the complete life cycle, placement analyses were performed at the most recent time in the cycle. In the second case each vintage was analyzed at different points in time as the experience of the vintage increased.

The IBFIT program generates average lives and dispersion patterns by comparing actual data to standard Iowa type survivor curves. The Iowa type survivor curves were derived empirically from the analyses of

TABLE 1. Placement mortality analysis on XBAR account of company CB

| Placement band | Account: Crossbar (1940-1983) Iowa type curve | ASL (years) |
|----------------|--|-------------|
| 1940 - 44 | R 2.5 | 42.3 |
| 1942 - 46 | R 4 | 38.0 |
| 1945 - 49 | R 4 | 31.7 |
| 1947 - 51 | R 3 | 32.3 |
| 1950 - 54 | R 3 | 29.9 |
| 1952 - 56 | R 4 | 27.5 |
| 1955 - 59 | R 4 | 25.4 |
| 1957 - 61 | L 3 | 25.7 |
| 1960 - 64 | L 3 | 21.2 |
| 1962 - 66 | L 3 | 19.4 |
| 1965 - 69 | S 2 | 16.0 |
| 1967 - 71 | S 1 | 14.2 |
| 1970 - 74 | L 2 | 11.7 |
| 1972 - 76 | L 2 | 10.6 |
| 1975 - 79 | S 0.5 | 8.6 |
| 1977 - 81 | S 0 | 7.3 |
| 1980 - 83 | L 1 | 5.8 |

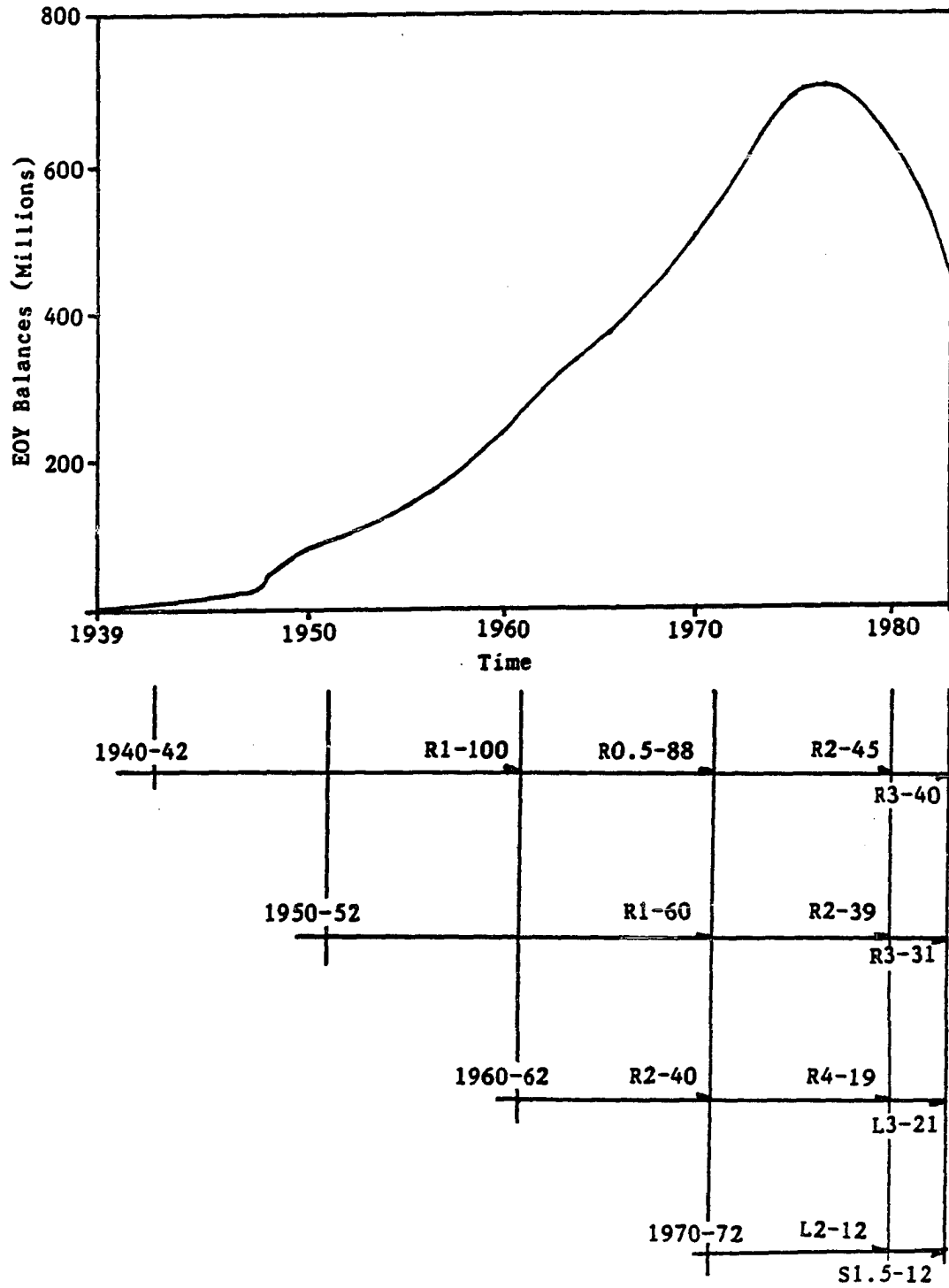


FIGURE 13. Placement mortality analysis with increasing experience

actual retirement data. The empirical nature of these type curves precludes mathematical combination of survivor curves with life cycles based on the observations mentioned above. However, the substantial consistency evolving from the processes underlying each observation can be easily explained.

The first observation about the declining trend in the average life indicators from old to new vintages is very much in tune with the explanation discussed in the turnover methods. When the means of producing a service, or a process is new, retirements from the early investments are quite negligible in spite of teething problems. The growing investment trend enhanced by the lack of significant retirements. The initial investments in the account are more like the foundation blocks on which the account grows. This gives a longer life span to the early placements. Apart from wear and tear, when capacity inadequacy sets in, similar life characteristics may be passed on to the replacements. However, when faced with technical and functional inadequacies, the newer placements are exposed to the same retirement policies as are the older vintages. By the virtue of the fact that older investments have survived to the point where the whole account is exposed to closure, the older vintages exhibit a longer life than the new ones.

Right modal survivor curves take their classification because the age of peak retirements for these curves occur after the average service life. This also means that there are more units surviving in the period prior to the average life line. Older vintages display

right modal characteristics; the reasons being as mentioned earlier that these placements have insignificant retirements until suddenly exposed to functional and capacity inadequacies. The newer investments are already in the technological progress environment. Retirement policies cause higher retirements from these investments before they attain their average life, thereby giving a left modal characteristics to these retirements.

The third observation arises from the amount of data points available for analysis. Few retirements, especially for the early vintages, for a long period of time make it difficult to analyze the type of pattern they may follow. Without further subjective input and any knowledge of future conditions, the information from life analysis can only help to estimate long service lives. The increase of experience with more retirements gives a more definite shape to the analysis. Though at times the patterns may be misleading, by and large the average life has a more definite estimate as the experience increases. The higher number of data points leaves it less to subjective analysis and more to objective interpretations. Based on this observation, one can note the difficulty of justifying a shorter life for a vintage during its earlier stages, in spite of the vintage finally providing a smaller amount of service.

Going through these observations or empirical regularities, one cannot help noticing the inter-dependency they exhibit with one another. At the same time they are related to the experience of the account which defines its life cycle. The combination of these

observations could as well be one of the nonpolitical causes for the huge depreciation reserve deficit.

V. INVESTMENT LIFE CYCLE MODEL

A product life cycle usually gives the functional relationship between the sales of a product, the dependent variable, and time, the independent variable. Although time is the principal variable in explaining product life cycles, several factors within the firm and the market are of relevance to the life cycle, very few of which are measurable [6]. Dean [13] maintains the length of the life cycle and its stages is a function of technical change, rate of market acceptance, and ease of competitive entry. The theory of diffusion and adoption of innovation is the basis for the theoretical rationale behind the product life cycle concept [39].

A. The Diffusion Process and Model

The diffusion process is defined as the spread of innovations to the members of a social system [40]. The diffusion process is distinguished from the adoption process which refers to the sequence of stages through which the adoption unit progresses from first awareness to final acceptance. Thus, having accepted the new item, diffusion is the movement of the innovation from adopter to adopter.

To develop the time pattern model of the diffusion process, Mahajan and Schoeman [30] introduce the following notation:

$f(t)$ = proportion of adopters at time t ;

$F(t)$ = cumulative proportion of adopters at time t .

At any time t

$$f(t) = dF(t)/dt.$$

The rate of diffusion at any time is assumed to be directly proportional to the proportion of potential adopters available at that time; that is, as the cumulative proportion of adopters approaches its maximum, say F^* , the rate of diffusion decreases proportionately. Mathematically, this can be written as:

$$f(t) = dF(t)/dt \text{ is proportional to } (F^* - F(t)).$$

Conceptually, this statement is representative of some engineering transfer equations, and population growth models.

The above statement requires a proportionality measure to relate the diffusion rate and the potential proportion of adopters. Introducing $g(t)$ as the constant of proportionality, we have:

$$f(t) = dF(t)/dt = g(t)(F^* - F(t)).$$

This is the rate equation for the diffusion process to be solved for $F(t)$.

The "constant" of proportionality $g(t)$ is a function of several parameters and it is more convenient to call it as the coefficient of diffusion. For a given innovation the value of $g(t)$ depends on the use of effective channels, and social system attributes. The perceived payoff or reward, and the size of investment or commitment of resources are also catalytic, affecting $g(t)$ [4]. As $(F^* - F(t))$ represents the proportion which has not yet adopted, the product $g(t)(F^* - F(t))$ give the expected proportion of adopters at time t . The coefficient of diffusion $g(t)$ may be interpreted as the probability of an adoption at time t .

Although it has been suggested that $g(t)$ can be represented as a

function of time, most researchers have worked with $g(t)$ as a function of the number of previous adopters. Expressing $g(t)$ as a function of $F(t)$ such as

$$g(t) = a + bF(t)$$

where a and b are model parameters, we get the mixed-influence model

$$dF(t)/dt = (a + bF(t))(F^* - F(t)) \quad [28].$$

The parameter a is defined as a coefficient of external influence. It is interpreted as representing external communications channels, mass media, government agencies, salespersons, etc. As such a can be considered to represent structured or formal channels of communications. The coefficient of internal influence b reflects the interaction of earlier adopters $F(t)$ with potential adopters ($F^* - F(t)$). In contrast to a , b represents unstructured, informal channels of communications. The presence of both parameters in the mixed-influence model makes it the most general version of the model [28].

B. Assumptions and the Investment Life Cycle Model

Against this background of the diffusion model, the life cycle for an account or the investment life cycle was developed. As an account represents year by year status of the investment in a particular form of plant or equipment, it is convenient to call this an account or investment life cycle as opposed to product life cycle. As explained in the earlier chapter, $B(t)$ the resultant end of the year balances were deemed to be the measure quantity for the life cycle. The life cycle model was developed on this criterion. While $F(t)$ represents the

cumulative proportion of adopters at time t , $B(t)$ is the cumulative number of survivors of the prior vintages. As opposed to $f(t) = dF(t)/dt$ which is the proportion of adopters at time t , $dB(t)/dt$ is the rate of change of the balances. The rate of change of the balances may be positive or negative depending on whether the additions in that year are more or less than the retirements respectively.

Some underlying assumptions are to be recognized before the model is developed.

1. It is decided either to invest in a particular type of plant or not to develop any account at all. Once this binary characteristic is overcome, $B(t)$ although measured in discrete end of the year amounts is a continuous process and its derivative exists at all points.
2. An account cannot be closed overnight initially. Once it is decided to open an account for a type of investment, it is very much into the growth stage.
3. There is an upper limit B^* on the amount of investment resulting from the additions and retirements in an account. Thus, $B(t)$ is unimodal; if the maximum amount continues for some time then the mode occurs at the midpoint of that continuity. Should there be a dip in the continuity the last occurrence decides the mode.
4. It is assumed that the type of plant represented by the account does not change over the life cycle. Minor modifications may take place but the basic principle

remains.

5. There is easy availability of information within and without the account as represented by the term $B(t)(B^* - B(t))$. And lastly, all relevant information about the process has been "captured" by the model.

In the telephone industry, as in any industry, much thought goes into an investment. Once the decision to invest has been made, the additions and/or retirements to the account are continuous throughout the year; this is evident from the day to day work orders. The binary effect (assumption 1) is based on the fact that some of the companies decided against investing in a particular type of switching equipment. Unless faced with some calamity, it would be imprudent to cancel a system soon after it shows up in the books. For a given community there is a limit on the number of subscribers to a phone system. Although as the demand increases and the investment grows, several external and internal factors constrain the growth. As per the definition of an account, it contains items having similar functional characteristics. Information about recent developments within the investment itself and also about competitive investments is readily available; this influences the status of the account. This is the closest that reality complies with the assumptions.

The general configuration of a typical investment life cycle is similar to an inverted parabola (Fig. 14). Although a product life cycle is usually divided into four stages, the investment life cycle can be partitioned into two sections to simplify discussion: the

growth section and the decline section. The rate of growth, or the rate of change to generalize, is positive in the growth section, and negative in the decline section. Thus, if the peak occurs at unity on the abscissa, the sign of the rate of change depends on its location with respect to the peak; mathematically, it depends on $(1 - t)$ where t is the proportional time unit.

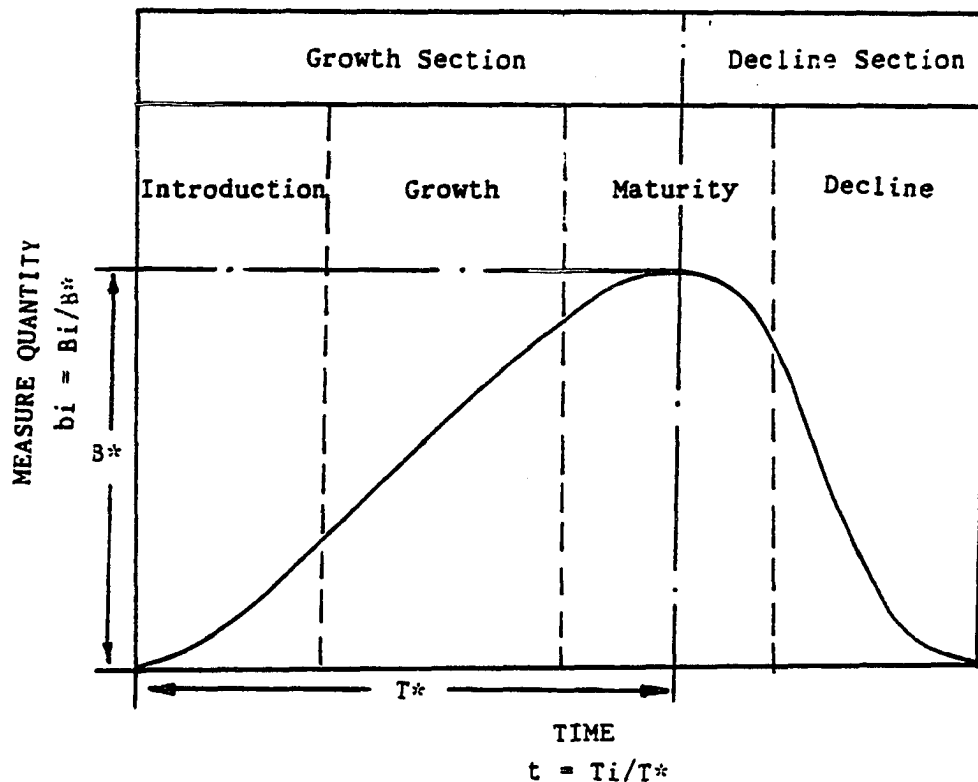


FIGURE 14. Life cycle sections for model development

As the life cycle theory emerges from that of the diffusion process, the rate of change is proportional to the amount that can yet be accommodated within the system. If B^* is the amount that a system

can accept, $(B^* - B(t))$ is the amount from the maximum value. The rate of change $dB(t)/dt$ is therefore proportional to $(B^* - B(t))$.

From above, we have the value of $dB(t)/dt$ is related to the product $(B^* - B(t))(1 - t)$, where $B(t)$ is the unit of measure of the life cycle or the resultant balance at time t . Thus, we have

$$dB(t)/dt \text{ is proportional to } (B^* - B(t))(1 - t).$$

If $b(t)$ is the portion resultant investment, $b(t) = B(t)/B^*$. Then

$$db(t)/dt = (1/B^*)(dB(t)/dt) \text{ or } dB(t)/dt = (B^*)(db(t)/dt).$$

Substituting this in the above proportionality, it becomes

$$(B^*)(db(t)/dt) \text{ is proportional to } (B^* - B(t))(1 - t)$$

or

$$db(t)/dt \text{ is proportional to } (1 - b(t))(1 - t).$$

If $g(t)$ is used to represent a "constant" of proportionality, the above relation may be rewritten as

$$db(t)/dt = g(t)(1 - b(t))(1 - t).$$

It is hypothesized that the value of $g(t)$ depends on the portion resultant investment and some function of time. This allows $g(t)$ to be written as

$$g(t) = b(t)(G(t))$$

where $G(t)$ is some function of time with respect to t . Substituting this in the earlier equation, we have

$$db(t)/dt = b(t)(1 - b(t))(1 - t)(G(t)) \quad (3)$$

To retain parsimony, and maintain the credibility induced by the term $(1 - t)$, $G(t)$ is kept simple. If $H(t)$ represents the integration of $(1 - t)(G(t))$, a solution of the above differential equation is

generalized as

$$\ln(b/(1 - b)) = H(t)$$

where the functional notation on b has been dropped.

C. Illustration

In order to check the validity of this life cycle model, investment data from different subaccounts from various telephone companies were analyzed. The subaccounts, from the central office equipment account, represent such switching equipment as manual switches, panel, step-by-step, crossbar, and electronic types. If not complete, the life cycles for some of these subaccounts had at least peaked and had entered the decline stage. The data were available by dollar amounts, and by units, that is, number of lines serviced by these switching equipment; although not for the same accounts. The telephone companies represented in the data are some Bell Telephone companies, and Centel Telephone companies.

The critical element for the acceptance of the data is the amount of completion of the life cycle. The data should represent investment made for the provision of service; and not an end product, for then it would be representative from a marketing point of view. The provision of service may be for use either within the company, like the use of a type of machine for the manufacture of a product, or without the company such as providing telecommunications or means of travel. The basic nature of the data, that is, investment representation, precludes its collection from organizational, and government publications. These

publications usually provide data on industry, national, or international levels. Individual companies need to be approached; even then the data may not be easily accessible, or broken down in a format from which can be made usable.

As explained earlier, the representative equation for the investment life cycle is:

$$db(t)/dt = b(t)(1 - b(t))(1 - t)(G(t)) \quad (3)$$

For solving this equation, it can be rewritten as:

$$db(t)/(b(t)(1 - b(t))) = (1 - t)(G(t))dt.$$

Integrating, we have the left hand side equal to

$$\ln(b/(1 - b)) \text{ dropping the functional notation on } b.$$

The solution to the right hand side, however, depends on the functional representation of $G(t)$. This functional representation should be simple, it should not influence the sign induction in the original equation, and it should be able to portray the decline stage of the life cycle. Some simple functions of $G(t)$ are presented in Table 2, along with the integration of $(1 - t)(Gt)$, $H(t)$. The coefficients p , q , r , and s under the $H(t)$ column are 1, 1/2, 1/3, and 1/4 respectively.

The solution to the differential Eq. (3) is

$$\ln(b/(1 - b)) = H(t) + c \quad (4)$$

and it represents the linear form for the investment life cycle. The Statistical Analysis System (SAS) package of computer programs, version 5, was utilized to test the model, and develop regression estimates of the parameters using end of the year balances for several accounts.

TABLE 2. Values of $G(t)$ and corresponding integrals $H(t)$

| $G(t)$ | $(1 - t)*G(t)$ | $H(t)$ |
|------------|---------------------|---------------------------|
| t | $t - t^2$ | $qt^2 - rt^3$ |
| $1 + t$ | $1 - t^2$ | $pt - rt^3$ |
| $t(1 + t)$ | $t - t^3$ | $qt^2 - st^4$ |
| t^2 | $t^2 - t^3$ | $rt^3 - st^4$ |
| $1 + t^2$ | $1 - t + t^2 - t^3$ | $pt - qt^2 + rt^3 - st^4$ |

The General Linear Model (GLM) procedure was used twice in analyzing the data. It was first used for the multiple linear regression of the linear form of the investment life cycle. Here the coefficient estimates for the independent variables generated in the right hand side, $H(t)$, were determined. It should be remembered that $H(t)$ was regressed against $\ln(b/(1 - b))$, and therefore, to check the fit against the original b , the GLM procedure was used the second time. Using the coefficient estimates generated in the earlier step, the linear expression was transformed to represent b in its original form. Using the GLM procedure as a simple linear regression the second time around, the model was regressed against the original data.

The model did not fit the data for all the versions of $G(t)$ presented in Table 2. The data appear to be in good agreement with the model where it is represented by:

$$G(t) = t^2 \quad \text{or} \quad G(t) = (1 + t^2).$$

Tables 3 and 4 display the regression results. Table 3 gives the values of B^* , and T^* along with the coefficient estimates for the equation

$$\ln(b/(1 - b)) = a + bt^3 + ct^4$$

for the corresponding data sets. Table 4 presents the coefficient estimates for the equation

$$\ln(b/(1 - b)) = a + bt + ct^2 + dt^3 + et^4.$$

The R^2 (coefficient of determination) values in both the cases for the second GLM procedure application are indicative of a good fit. However, all the coefficient estimates for $G(t) = t^2$ are statistically significant, even though the R^2 values are not as high as those for $G(t) = (1 + t^2)$. One cannot reject the null hypothesis that some coefficients may be equal to zero in the later case. The signs of the coefficient estimates are not always true to the model. The inconsistency of the coefficient estimates developed from $G(t) = (1 + t^2)$ is the problem of near-confounding of effects where independent variables are highly related [5]. Figures 15 and 16 show the actual values of investment (dollars or lines in service) and the values by the regression equation for some of the subaccounts analyzed. For every subaccount studied the model regression equation describes the general trend of the time path of the life cycle very well. It should be noted that the investment life cycle is represented by

$$b = \exp(fx)/(1 + \exp(fx))$$

where fx is the right hand side of Eq. (4).

TABLE 3. Coefficients for $G(t) = t^2$

| Com- pany | Account | B* | T* | a |
|--------------|---------|------------|----|-------------|
| CF | SXS(\$) | 37659978 | 30 | -3.6109765 |
| CI | SXS(\$) | 35259438 | 23 | -3.3721483 |
| CM | SXS(\$) | 8717943 | 35 | -2.5777816 |
| CN | SXS(\$) | 30528322 | 32 | -3.4647021 |
| CV | SXS(\$) | 33628151 | 33 | -3.2852743 |
| CB | MAN(\$) | 3291091 | 69 | -3.1665318 |
| CB | PAN(\$) | 16422090 | 40 | -2.2980123 |
| CB | SXS(\$) | 9289407 | 38 | -3.6432797 |
| CB | XBA(\$) | 80998843 | 37 | -3.1876617 |
| CB | ESS(\$) | 1175000000 | 22 | -2.0107166 |
| IB | PAN(\$) | 38669168 | 30 | 9.7485651 |
| IB | SXS(\$) | 41918233 | 36 | -7.6491489 |
| IB | XBA(\$) | 706470651 | 38 | -3.1456162 |
| ID | SXS(\$) | 152961698 | 40 | -3.4194211 |
| ID | XBA(\$) | 111261819 | 30 | -4.4451768 |
| MB | PAN(L) | 288271 | 30 | -0.1018122 |
| MB | SXS(L) | 878850 | 39 | -2.1671837 |
| MB | XBA(L) | 1614097 | 34 | -2.6244366 |
| MB | ESS(L) | 10000000 | 23 | -4.6411115 |
| OB | MAN(\$) | 46287038 | 61 | -4.9401807 |
| OB | PAN(\$) | 29022823 | 43 | -1.4973589 |
| OB | SXS(\$) | 167799460 | 40 | -2.3068812 |
| OB | XBA(\$) | 299568909 | 39 | -3.8031575 |
| OB | ESS(\$) | 900000000 | 27 | -6.8870331 |
| PB | PAN(L) | 203 | 37 | -1.6731708 |
| PB | SXS(L) | 2557 | 38 | -1.28716468 |
| PB | CDO(L) | 290 | 43 | -1.5495646 |
| PB | XBI(L) | 575 | 39 | -2.36022104 |
| PB | XBS(L) | 4119 | 31 | -3.7791483 |

| b | c | R ² | R* ² | C* |
|-------------|--------------|----------------|-----------------|------|
| 26.4303999 | -13.5039523 | .360064 | .881073 | 0.65 |
| 14.8479248 | -9.4437458 | .30948 | .70681 | 0.75 |
| -2.0621816 | 16.1533510 | .677175 | .897465 | 0.65 |
| 7.9953754 | 2.9224823 | .590854 | .98221 | 0.7 |
| 17.9479286 | -9.9870677 | .466531 | .971962 | 0.65 |
| 21.3908514 | -16.9078539 | .193561 | .30316 | 0.7 |
| 35.1114591 | -28.9875128 | .384745 | .890159 | 0.5 |
| 27.1847068 | -20.3177262 | .422456 | .882321 | 0.65 |
| 26.6109106 | -19.8859362 | .399264 | .905346 | 0.6 |
| 32.2534599 | -31.9227853 | .93111 | .973756 | 0.55 |
| -8.7512118 | 3.9510637 | .213522 | .672314 | 0.9 |
| 60.0751349 | -47.829599 | .346992 | .851127 | 0.65 |
| 24.2024378 | -17.2774339 | .488991 | .941718 | 0.65 |
| 26.5186002 | -20.1736479 | .3223 | .862557 | 0.65 |
| 30.16134854 | -22.2127708 | .422 | .887059 | 0.65 |
| 15.6624923 | -10.8018589 | .540721 | .920629 | 0.45 |
| 26.7449068 | -21.0606872 | .348144 | .889058 | 0.55 |
| 33.3854937 | -26.5880992 | .51258 | .940213 | 0.55 |
| 53.0443311 | -61.5091056 | .907855 | .885142 | 0.65 |
| 22.7771989 | -16.3443141 | .276314 | .592459 | 0.75 |
| 32.2825067 | -28.2150898 | .178263 | .850129 | 0.45 |
| 26.522949 | -20.4660265 | .316454 | .925253 | 0.55 |
| 26.4551092 | -18.9274666 | .435808 | .945120 | 0.65 |
| 107.2315381 | -121.5148765 | .715415 | .756887 | 0.75 |
| 28.5455005 | -22.3364647 | .77621 | .984634 | 0.5 |
| 20.5788298 | -15.4836433 | .674611 | .872992 | 0.5 |
| 21.6634769 | -17.2859005 | .645454 | .950924 | 0.55 |
| 35.70088407 | -29.3472686 | .653258 | .901113 | 0.5 |
| 27.4770034 | -20.248381 | .677407 | .949204 | 0.65 |

TABLE 3. Continued

| Com- pany | Account | B* | T* | a |
|--------------|---------|-----------|----|------------|
| PB | ESS(L) | 9120 | 24 | -4.5484816 |
| SC | SXS(L) | 1693316 | 36 | -4.6143529 |
| SC | CDO(L) | 1549230 | 20 | 4.3733877 |
| SC | XBA(L) | 2171285 | 39 | -6.6138563 |
| SC | ESS(L) | 5000000 | 23 | -5.5403434 |
| SN | MAN(\$) | 9317244 | 60 | -4.5162936 |
| SN | SXS(L) | 188622230 | 40 | -2.9948412 |
| SN | XBA(L) | 98645970 | 26 | -3.6568642 |
| SN | ESS(L) | 600000000 | 24 | -4.1975977 |
| SW | PAN(L) | 277 | 37 | -0.7732287 |
| SW | SXS(L) | 3725 | 52 | -3.7016622 |
| SW | XBI(L) | 348 | 31 | -2.9554559 |
| SW | XBS(L) | 2192 | 25 | -3.1338865 |
| SW | ESS(L) | 10000 | 22 | -4.1324072 |
| WB | PAN(L) | 58514 | 40 | -3.0990113 |
| WB | SXS(L) | 450144 | 39 | -3.0206319 |
| WB | XBA(L) | 527287 | 33 | -2.374709 |
| WB | ESS(L) | 1500000 | 22 | -3.2333596 |
| P | B707(#) | 117 | 11 | 0.1190689 |

| b | c | R ² | R* ² | C* |
|-------------|-------------|----------------|-----------------|------|
| 27.9100854 | -17.7929589 | .895501 | .987267 | 0.65 |
| 33.4743748 | -24.3386043 | .414932 | .779398 | 0.65 |
| -3.0707537 | 1.21024378 | .101001 | .004317 | 0.8 |
| 41.0093634 | -30.9708337 | .370102 | .917016 | 0.7 |
| 82.6454377 | -89.7005277 | .92482 | .954584 | 0.55 |
| 26.6179825 | -18.7105573 | .389313 | .779252 | 0.7 |
| 26.24453108 | -20.1717624 | .303304 | .87694 | 0.6 |
| 36.4891637 | -27.8526264 | .435055 | .957544 | 0.55 |
| 63.00861688 | -69.5328867 | .953868 | .973306 | 0.6 |
| 22.5782531 | -18.3781344 | .683134 | .958564 | 0.45 |
| 26.0090091 | -19.5717958 | .709548 | .941025 | 0.65 |
| 33.5049444 | -26.4356941 | .781577 | .931129 | 0.55 |
| 23.0521405 | -16.317133 | .639812 | .896724 | 0.3 |
| 64.6862995 | -69.3196598 | .915854 | .959036 | 0.55 |
| 43.2684153 | -36.1818077 | .325203 | .480871 | 0.5 |
| 27.8586944 | -21.584136 | .410634 | .853376 | .06 |
| 33.3414864 | -27.1471599 | .472382 | .906198 | 0.55 |
| 43.5129575 | -51.7516395 | .935814 | .979670 | 0.65 |
| 1.7722751 | -0.931637 | .279695 | .613664 | 0.65 |

TABLE 4. Coefficients for $G(t) = 1 + t^2$

| Com- pany | Account | a | b | c |
|--------------|---------|---------------|--------------|---------------|
| CF | SXS(\$) | -10.0358883 | 81.14163004 | -274.0108009 |
| CI | SXS(\$) | -5.7944874 | -3.62451998 | 45.2030753 |
| CM | SXS(\$) | -1.9544796 | -47.2754353 | 278.8515262 |
| CN | SXS(\$) | -5.4862725 | 9.6446049 | 8.9594875 |
| CV | SXS(\$) | -8.2710673 | 59.4262142 | -202.8198182 |
| CB | MAN(\$) | 242.9061863 | -1064.998240 | 1682.6646942 |
| CB | PAN(\$) | -25.6712708 | 183.4846544 | -466.395311 |
| CB | SXS(\$) | -10.8163157 | 65.7593503 | -181.0846318 |
| CB | XBA(\$) | -6.8392146 | 54.0761608 | -190.6986594 |
| CB | ESS(\$) | -4.0127005 | 26.0362487 | -94.3004958 |
| IB | PAN(\$) | -1718.9629659 | 5685.1345959 | -6913.7752241 |
| IB | SXS(\$) | 9.0683961 | 67.1356065 | -381.9943398 |
| IB | XBA(\$) | -5.8876112 | 44.0074942 | -163.4364022 |
| ID | SXS(\$) | -10.5572322 | 78.9136275 | -248.3941493 |
| ID | XBA(\$) | -7.8813068 | 49.3495211 | -173.4260978 |
| MB | PAN(L) | -47.9919142 | 200.7850815 | -302.9749679 |
| MB | SXS(L) | -3.9513193 | 26.8904444 | -91.1730623 |
| MB | XBA(L) | -5.5456722 | 47.3790777 | -175.3140415 |
| MB | ESS(L) | -6.1095079 | 10.5349913 | 1.8261381 |
| OB | MAN(\$) | -138.3309252 | 459.5153302 | -571.6768865 |
| OB | PAN(\$) | -80.2503384 | 563.7331884 | -1388.2440847 |
| OB | SXS(\$) | -8.5218388 | 70.4762751 | -224.8539733 |
| OB | XBA(\$) | -7.2882334 | 50.9562565 | -183.1942444 |
| OB | ESS(\$) | -15.7146888 | 99.5661094 | -292.6336351 |
| PB | PAN(L) | 11.9712328 | -73.0870946 | 137.2769598 |
| PB | SXS(L) | 3.0957314 | -41.9400933 | 112.7882442 |
| PB | CDO(L) | -3.1635288 | 25.5604713 | -93.4477518 |
| PB | XBI(L) | -7.0114332 | 53.6634004 | -163.4243747 |
| PB | XBS(L) | -7.9099232 | 34.2172658 | -76.7601239 |

| d | e | R ² | R* ² | C* |
|---------------|---------------|----------------|-----------------|------|
| 360.1010798 | -152.353225 | .432035 | .930311 | 0.7 |
| -44.17095843 | 11.1967795 | .415142 | .813638 | 0.55 |
| -491.5211129 | 278.6691351 | .786548 | .907167 | 0.5 |
| -44.4440036 | 39.1926142 | .612884 | .99149 | 0.65 |
| 276.25200631 | -119.44708019 | .48485 | .973698 | 0.7 |
| -1131.4888004 | 272.7545273 | .308651 | .49687 | 0.6 |
| 503.8752353 | -190.492963 | .464841 | .916008 | 0.35 |
| 217.2337517 | -87.3858304 | .452148 | .922075 | 0.65 |
| 263.6425702 | -115.5908966 | .452013 | .917717 | 0.2 |
| 161.40 | -91.2820152 | .99778 | .99779 | 0.5 |
| 3677.5228406 | -723.4969159 | .285177 | .911351 | 0.85 |
| 548.8740167 | -237.4039976 | .426104 | .866177 | 0.55 |
| 235.8389408 | -105.8475256 | .529283 | .933366 | 0.65 |
| 315.4352849 | -131.2694969 | .379157 | .882069 | 0.7 |
| 246.9210932 | -110.5769989 | .450673 | .907552 | 0.7 |
| 211.0109874 | -56.3622795 | .545195 | .925886 | 0.5 |
| 134.6563293 | -62.4348081 | .369486 | .843955 | 0.25 |
| 258.2890908 | -119.5824134 | .569808 | .931879 | 0.25 |
| -18.5331712 | 12.0469207 | .997635 | .998345 | 0.65 |
| 326.7306858 | -74.4261341 | .289697 | .650133 | 0.8 |
| 1438.5803531 | -528.7805583 | .369525 | .778918 | 0.35 |
| 290.1108169 | -122.3289216 | .370670 | .861962 | 0.3 |
| 261.74190188 | -117.7549443 | .468523 | .946126 | 0.65 |
| 403.2300923 | -202.5370247 | .983659 | .990469 | 0.6 |
| -79.9998885 | 8.4590689 | .782803 | .9863 | 0.3 |
| -91.0192876 | 20.95585203 | .716946 | .913402 | 0.1 |
| 139.6952692 | -65.2053813 | .700614 | .951588 | 0.2 |
| 220.5754696 | -99.3320695 | .733301 | .965665 | 0.3 |
| 92.7909323 | -39.1501056 | .737883 | .968672 | 0.55 |

TABLE 4. Continued

| Com- pany | Account | a | b | c |
|--------------|---------|---------------|---------------|--------------|
| PB | ESS(L) | -7.8149101 | 15.5637876 | 16.2297777 |
| SC | SXS(L) | 237.4831705 | -1583.5581479 | 2658.7945693 |
| SC | CDO(L) | 338.2961998 | -806.6916061 | -703.7438885 |
| SC | XBA(L) | -131.69101759 | 744.9230543 | -158.9755971 |
| SC | ESS(L) | -8.5178845 | 26.2234172 | -43.2176023 |
| SN | MAN(\$) | 713.2504432 | -2966.075322 | 4489.6287211 |
| SN | SXS(L) | -8.9926333 | 68.4699794 | -219.2133774 |
| SN | XBA(L) | -11.7963595 | 105.0104644 | -349.9676067 |
| SN | ESS(L) | -5.5023386 | 8.9252455 | 4.44084667 |
| SW | PAN(L) | -2.6947837 | 19.2675648 | 51.7198969 |
| SW | SXS(L) | -6.1989597 | 32.6642203 | -104.3609895 |
| SW | XBI(L) | -5.6813737 | 33.3471009 | -103.5437225 |
| SW | XBS(L) | -5.4333809 | 16.5894973 | -24.2882625 |
| SW | ESS(L) | -6.4129315 | 16.7842086 | -12.0939298 |
| WB | PAN(L) | -196.3610153 | 1167.3072888 | -2534.547682 |
| WB | SXS(L) | -5.5879214 | 22.5384539 | -61.4394859 |
| WB | XBA(L) | -7.8204883 | 65.4959034 | -208.8772629 |
| WB | ESS(L) | -5.4005046 | 22.5465949 | -58.3381587 |
| P | B707(#) | -1.0119174 | -10.0604712 | 34.9006413 |

| d | e | R ² | R* ² | C* |
|---------------|--------------|----------------|-----------------|------|
| -63.95803041 | 46.9206617 | .986392 | .991730 | 0.6 |
| -1886.8002616 | 480.0471844 | .541221 | .857113 | 0.55 |
| -267.1048967 | 37.3158347 | .331057 | .277592 | 0.9 |
| 1471.9937855 | -495.5034656 | .438252 | .922983 | 0.75 |
| 62.5996014 | -38.4987068 | .997968 | .9988 | 0.55 |
| -2924.2881599 | 692.5832332 | .541609 | .932737 | 0.7 |
| 283.7302054 | -119.8014514 | .353249 | .867871 | 0.65 |
| 461.3535315 | -198.1835071 | .531064 | .923784 | 0.65 |
| -4.8931115 | -2.5698719 | .995450 | .997105 | 0.55 |
| 74.7190667 | -36.1081723 | .712779 | .981518 | 0.25 |
| 145.1736943 | -64.0993133 | .752445 | .931188 | 0.65 |
| 149.9699951 | -69.5877394 | .816092 | .941076 | 0.5 |
| 31.5672851 | -15.2477066 | .724216 | .944069 | 0.5 |
| 12.7100229 | -11.8733671 | .996756 | .998547 | 0.5 |
| 2391.0479744 | -821.615871 | .499502 | .793641 | 0.5 |
| 92.3570523 | -44.4370731 | .414546 | .838514 | 0.55 |
| 275.5251156 | -119.8387569 | .563534 | .917372 | 0.3 |
| 93.5381587 | -52.891233 | .998092 | .997751 | 0.5 |
| -26.9072848 | 5.9199983 | .629497 | .88618 | 0.5 |

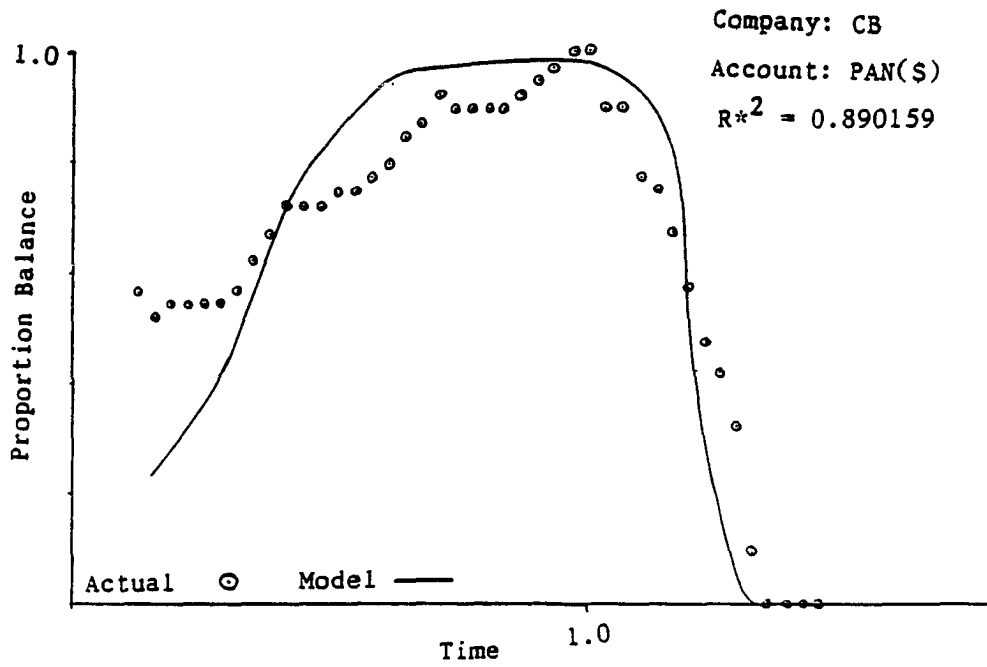
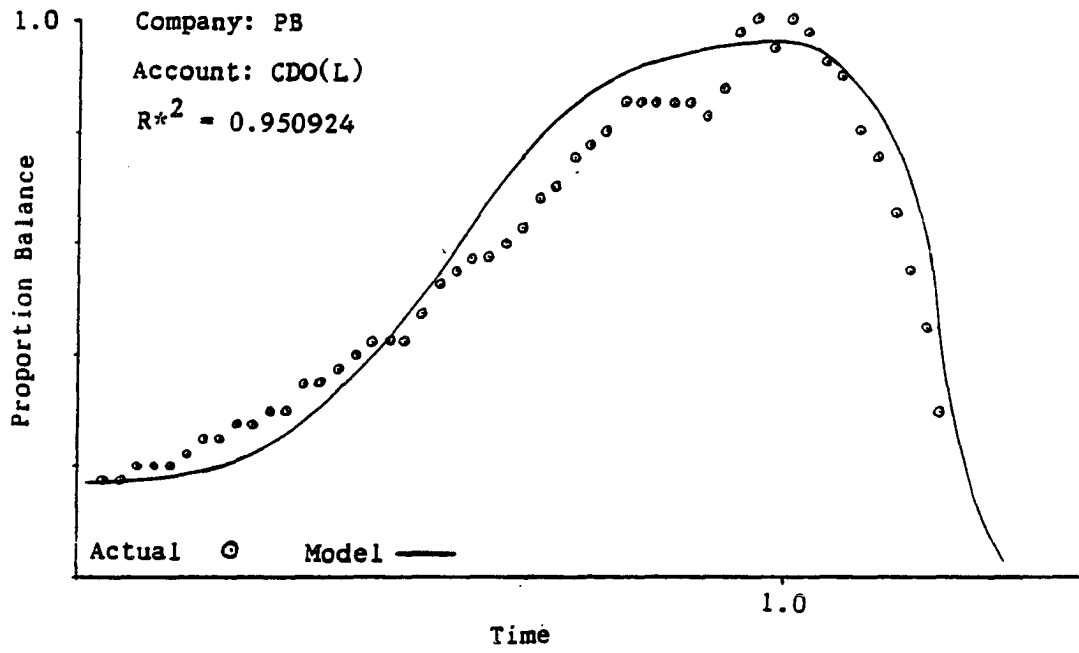


FIGURE 15. Fitting for $G(t) = t^2$

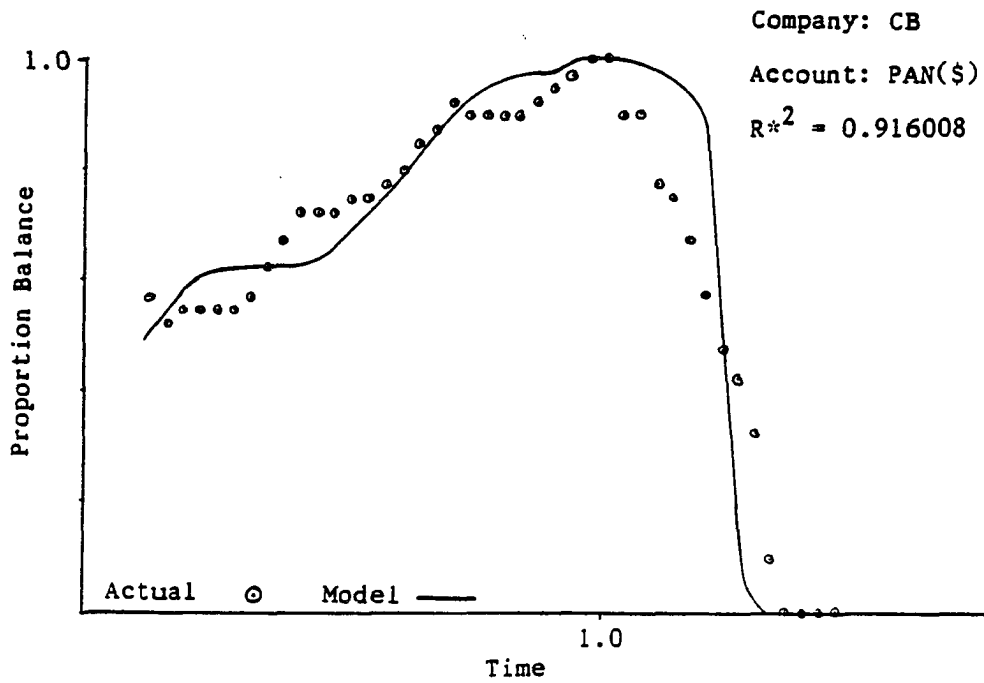
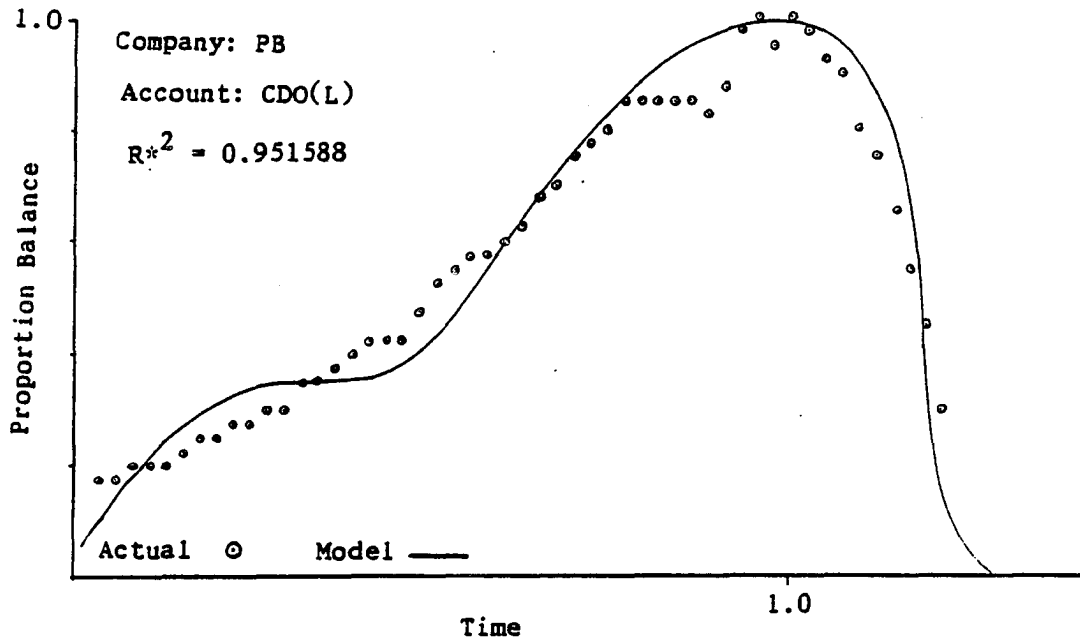


FIGURE 16. Fitting for $G(t) = 1 + t^2$

VI. DEVELOPMENT OF LIFE CYCLE CURVE TYPES

The application of the product life cycle concept to capital recovery was introduced as recently as 1979 [35]. As in life cycle costing, the basic philosophy herein is that capital should be recovered by the time the plant is fully retired. However, as explained earlier, the service life indicators are quite high in the initial stages of the life cycle and placement as well. The low depreciation rates induced by the high life indication cause insufficient accumulation of depreciation when the high retirement rates set in. Correcting the depreciation rates on a whole life basis to accommodate the later high retirement pattern does not necessarily result in total recovery of capital unless amortization of the earlier deficit is allowed. Amortization, although allowing for complete recovery of investment, raises the question about proper allocation of the cost of providing service duly to the customers.

The recent acceptance of using remaining life procedure is a more appropriate alternative solution to this problem. Using classical retirement ratio theory, Johnson [26] has proposed the use of forecasts of additions and retirements from marketing and construction divisions of a company to compute retirement ratios for a given account. Assuming future retirements to be independent of age, the remaining life for the embedded plant can be determined by applying these retirement ratios.

Mathematically, this assumption causes the future retirements to be prorated with respect to the balances of the embedded plant and the

future additions. Retirement ratio is the ratio of the amount of retirements during a period to the amount exposed for retirement at the beginning of the period. Thus

$$RR_i = R_i/B(i - 1).$$

If $EP(i - 1)$ is the amount of embedded plant in the previous year's balance, then according to Johnson the amount of retirements from the embedded plant is:

$$R(EP) = RR_i * EP(i - 1).$$

Now as

$$R(EP) = \{R_i/B(i - 1)\} * EP(i - 1),$$

therefore

$$R(EP) = R_i * \{ EP(i - 1)/B_i \},$$

that is, the retirements are prorated in the ratio of survivors of embedded plant to total balances.

The reliability of this method depends on the confidence placed on the forecasts and their duration. It gives dependable results as long as the forecasts are for a short period. Implicitly, this means that the account should be very much in the decline stage of its life cycle, and the forecasts are for the remaining few years of the life span. In such a case one may also rely on the prior information available in much detail to compute the remaining lives based on traditional life analysis techniques.

Along with a similar approach using retirement ratios, Ocker [35] presents the idea of an overall life for capital recovery. In his paper he has argued that by comparing the historical life cycle of one

technology to the life cycle of an existing technology and assuming a similar growth rate, life span, and retirement pattern, it is possible to estimate an average life for the new. Although this philosophy that plant being installed or in general use bears resemblance to plant which has been or is being retired is similar to that in traditional life analysis and estimation techniques, and it is alike that presented by Clark [9], the choice of the comparable historical life cycle is not singular; Clark mentions a different technology for the basis. This disagreement may arise due to the different backgrounds of the authors. But to a third party, this is an area of concern: which historical technology should be taken as the unit of measure? An answer to this question is the development of type life cycle curves on a generalized basis.

A. Generalized Life Cycles

The model for the life cycle, presented in the earlier chapter, represents the historical, and existing technologies very well. Without pretension to the background of the technology, a generalized version of this model can be well applied to the life cycle of an existing technology. The model itself has been developed on a normalized basis for time, and unit of measure. Thus the curves are easily comparable irrespective of the number of years to reach the maximum amount as well as the maximum amount itself.

As with product life cycles, the life cycles generated by the investments produced unique patterns, in spite of the investments being

made in the same technology. The grouping of the curves on a technology basis was dropped. Most of the data sets studied did not peak in the same time interval; a few groups could be formed, but none of these groups contained any more than three data sets. The time interval required for an account to peak was found not to be useful for the grouping of the curves, neither was the peaking amount for the same reasons.

The model was developed from the differential equation representing the slope of the life cycle:

$$db(t)/dt = b(t)(1 - b(t))(1 - t)(G(t)) \quad (3)$$

The configuration of this equation was studied. Looking at a typical slope curve (Fig. 17), it can be seen that the rate of change of balances increases, and then decreases during the growth stage of the life cycle. A similar observation can be made for the decline part of the life cycle, however, in the negative quadrant. The occurrence of the positive peak ranged from about 0.2 to about 0.9 times the time interval required for the life cycle to reach its maximum. As the growth stage has a longer span than the decline stage (three times longer [35]), as the growth stage is more descriptive of the configuration of the life cycle the differential equation for the growth stage was opted for further investigation.

Recall from the earlier chapter that two values of the time function $G(t)$ give good fits of the model to the data; the R^2 values for $G(t) = (1 + t^2)$ are a bit higher than those for $G(t) = t^2$. However, as the number of interrelated terms increases the problem of

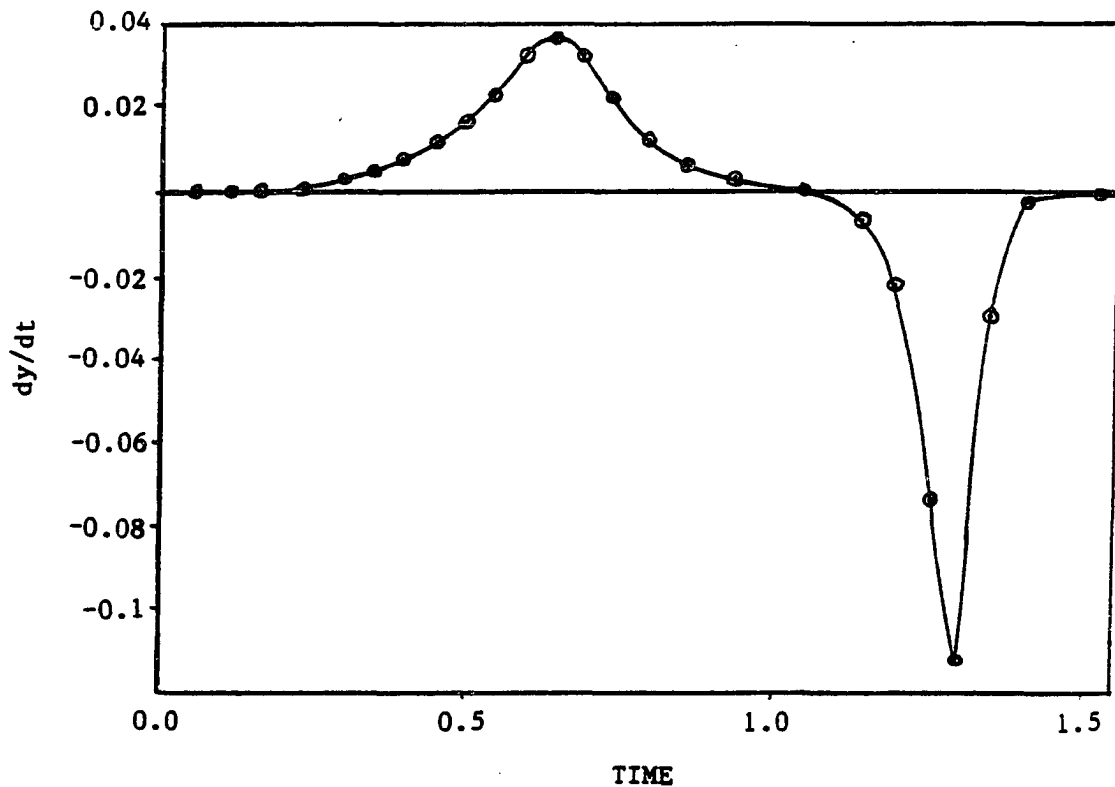


FIGURE 17. Typical slope curve for life cycle

near-confounding of effects especially in the first case [5]. This is evident as one cannot reject the null hypothesis that some of the coefficient estimates of the former case are equal to zero based on the regression analysis. Moreover, as the near-confounding of effects places the equation in a plane that tends to minimize the sum of the square errors, the coefficient estimates have large standard errors, and their values are widely dispersed. Accordingly $G(t) = t^2$ was deemed to be a better choice for generalizing the curves.

Using the coefficient estimates of all the data sets analyzed, the differential equation (I) for each one was traced. At constant

intervals of $t = 0.05$, the function was evaluated between $t = 0$, and $t = 1$. The peaks occurred at distinct points along the time scale. The occurrence of the peak change rate for each account analyzed is noted under column C* in Tables 3 and 4. Grouping the curves by the point of the occurrence of the peak resulted in the formation of seven groups. Some groups only contained a few curves and others contained as many as eight. The type curve for each group was obtained by averaging the coefficient estimates of the curves in the group. These parameter coefficients are presented in Table 5. The type life cycle rate curves from these coefficients are shown in Fig. 18.

TABLE 5. Coefficients for type life cycle curves with t^2

| C* | Curve | a | b | c |
|------|-------|------------|------------|-------------|
| 0.45 | H | -0.7907999 | 23.5077507 | -19.1316944 |
| 0.5 | C | -2.1435161 | 32.6410176 | -26.4673394 |
| 0.55 | K | -2.5478708 | 30.2360601 | -23.8337421 |
| 0.6 | O | -3.0871878 | 26.2291435 | -19.7298171 |
| 0.65 | J | -4.1589543 | 29.7254391 | -21.6464681 |
| 0.7 | T | -4.4403459 | 24.2533932 | -15.9166907 |
| 0.78 | D | -4.1561645 | 18.8125619 | -12.8940299 |

Although the multicollinearity engendered by $G(t) = (1 + t^2)$ causes the problem of near-confounding of effects, one cannot overlook the fact that the model from this expression has a comparatively better

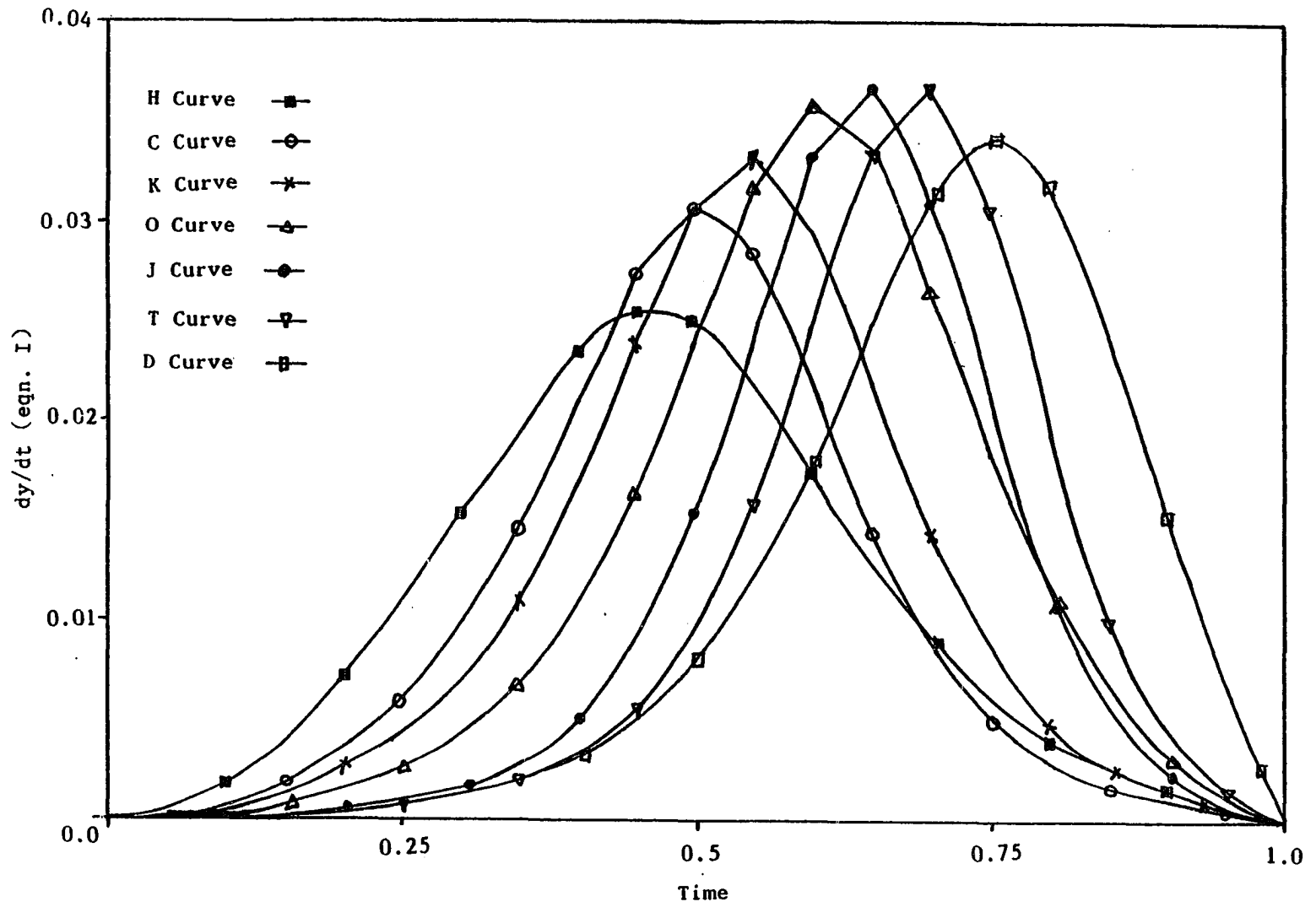


FIGURE 18. Type life cycle rate curves

fit. Using arguments similar to the case of $G(t) = t^2$, one can develop coefficient estimates for type life cycle curves for $G(t) = (1 + t^2)$. These are presented in Table 6.

The significance of these type curves is that they cover a certain range of physical property affected by technology and competition. However, the original equation can hold good for any other type of investment life cycle where the substitution of the investment occurs due to technical, and economical efficiencies. For example, Fig. 19 gives the actuarial data for the Boeing 707 (B707) account of an international airlines from United States. Using $G(t) = (1 + t^2)$, the equation fits the data quite well ($R^2 = 0.9$), as shown in Fig. 20. It should be pointed out that the shape of the life cycle in this case is not totally influenced by technology and competition as in the earlier case of the telephone industry.

What we have here is that a certain classification of curves being typified. This particular classification is based on certain technological advancements which may not be common to all. The classification is therefore not final nor conclusive; it is possible to extend the investment life cycle classifications by analyzing data facing different managerial, economic, and technical conditions.

B. Application

The generalized life cycle type curves have been derived without reference to any specific technical background of central office equipment, time scale, or quantifying units. By selecting a life cycle

TABLE 6. Coefficients of type life cycle curves for $g(t) = 1 + t^2$

| C* | Curve | a | b | c | d | e |
|------|-------|-------------|-------------|--------------|-------------|--------------|
| 0.25 | H | -4.4389037 | 34.6355437 | -120.4706784 | 174.2004652 | -79.7845343 |
| 0.3 | C | -19.5506894 | 143.9610545 | -385.7531349 | 441.4445168 | -175.3857001 |
| 0.5 | K | -5.0424851 | 25.3242823 | -74.0441603 | 114.3146826 | -58.7058204 |
| 0.55 | O | -6.7489223 | 28.3778599 | -69.0998049 | 92.5739923 | -41.7935894 |
| 0.65 | J | -8.6989536 | 54.0681634 | -164.7879179 | 216.1694381 | -93.8188958 |
| 0.7 | T | -9.1630796 | 67.2077482 | -224.6807575 | 299.6773661 | -128.4117003 |

Company P: Account for

| Year of Placement (vintage) | Installed during year | UPPER FIGURES: In service at LOWER FIGURES: Retired during | | | | | | | | | | | | |
|--------------------------------------|-----------------------------|---|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| | | '59 | '60 | '61 | '62 | '63 | '64 | '65 | '66 | '67 | '68 | '69 | '70 | '71 |
| 1959 | 15 | 15 | 15 | 15 | 15 | 15 | 15 | 15 | 15 | 15 | 15 | 15 | 15 | 12 |
| | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3 |
| 1960 | 8 | | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 7 |
| | | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 1961 | 3 | | | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 0 |
| | | | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3 |
| 1962 | 5 | | | | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 |
| | | | | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1963 | 3 | | | | | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 |
| | | | | | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1964 | 0 | | | | | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | | | | | | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1965 | 11 | | | | | | | 11 | 11 | 11 | 11 | 11 | 11 | 11 |
| | | | | | | | | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1966 | 10 | | | | | | | | 10 | 10 | 10 | 10 | 10 | 10 |
| | | | | | | | | | 0 | 0 | 0 | 0 | 0 | 0 |
| 1967 | 12 | | | | | | | | | 12 | 12 | 12 | 12 | 12 |
| | | | | | | | | | | 0 | 0 | 0 | 0 | 0 |
| 1968 | 15 | | | | | | | | | | 14 | 14 | 14 | 14 |
| | | | | | | | | | | | 1 | 0 | 0 | 0 |
| 1969 | 12 | | | | | | | | | | | 12 | 12 | 12 |
| | | | | | | | | | | | | 0 | 0 | 0 |
| TOTALS: | In Service End of Year | 15 | 23 | 25 | 31 | 34 | 34 | 45 | 55 | 67 | 81 | 93 | 93 | 86 |
| | Retired During Year | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 7 |

FIGURE 19. Actuarial data for B-707s

Account for Boeing 707 aircraft (B707)

URES: In service at end of indicated calendar year
 URES: Retired during calendar year

| | | | | | | | | | | | | | | | | | Average Service Life | |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|----------------------------|-------|
| '68 | '69 | '70 | '71 | '72 | '73 | '74 | '75 | '76 | '77 | '78 | '79 | '80 | '81 | '82 | '83 | '84 | '85 | |
| 15 | 15 | 15 | 12 | 9 | 6 | 6 | 5 | 2 | 1 | 0 | | | | | | | | 14.23 |
| 0 | 0 | 0 | 3 | 3 | 3 | 0 | 1 | 3 | 1 | 1 | | | | | | | | |
| 8 | 8 | 8 | 7 | 6 | 5 | 4 | 3 | 1 | 1 | 0 | | | | | | | | 13.88 |
| 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 2 | 1 | 1 | | | | | | | | |
| 3 | 3 | 3 | 0 | | | | | | | | | | | | | | | 9.5 |
| 0 | 0 | 0 | 3 | | | | | | | | | | | | | | | |
| 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 0 | | | | | | | | 15.5 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 5 | | | | | | | | |
| 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 0 | | | | | | | | | 13.5 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3 | | | | | | | | | |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 11 | 11 | 11 | 11 | 11 | 11 | 10 | 10 | 10 | 10 | 10 | 5 | 2 | 2 | 0 | | | | 13.86 |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 5 | 3 | 0 | 2 | | | | |
| 10 | 10 | 10 | 10 | 10 | 10 | 9 | 8 | 6 | 5 | 4 | 3 | 2 | 0 | | | | | 11.20 |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 1 | 1 | 1 | 1 | 2 | | | | | |
| 12 | 12 | 12 | 12 | 12 | 12 | 12 | 11 | 11 | 11 | 8 | 6 | 5 | 5 | 2 | 0 | | | 12.42 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 3 | 2 | 1 | 0 | 3 | 2 | | | |
| 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 11 | 11 | 11 | 10 | 8 | 8 | 7 | 7 | 6 | 0 | 12.30 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3 | 0 | 0 | 1 | 2 | 0 | 1 | 0 | 1 | 6 | |
| | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 10 | 10 | 10 | 10 | 10 | 9 | 6 | 5 | 0 | 13.33 |
| | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 0 | 0 | 0 | 0 | 1 | 3 | 1 | 5 | |
| 81 | 93 | 93 | 86 | 82 | 78 | 75 | 71 | 61 | 54 | 43 | 34 | 27 | 25 | 18 | 13 | 11 | 0 | |
| 1 | 0 | 0 | 7 | 4 | 4 | 3 | 4 | 10 | 7 | 11 | 9 | 7 | 2 | 7 | 5 | 2 | 11 | |

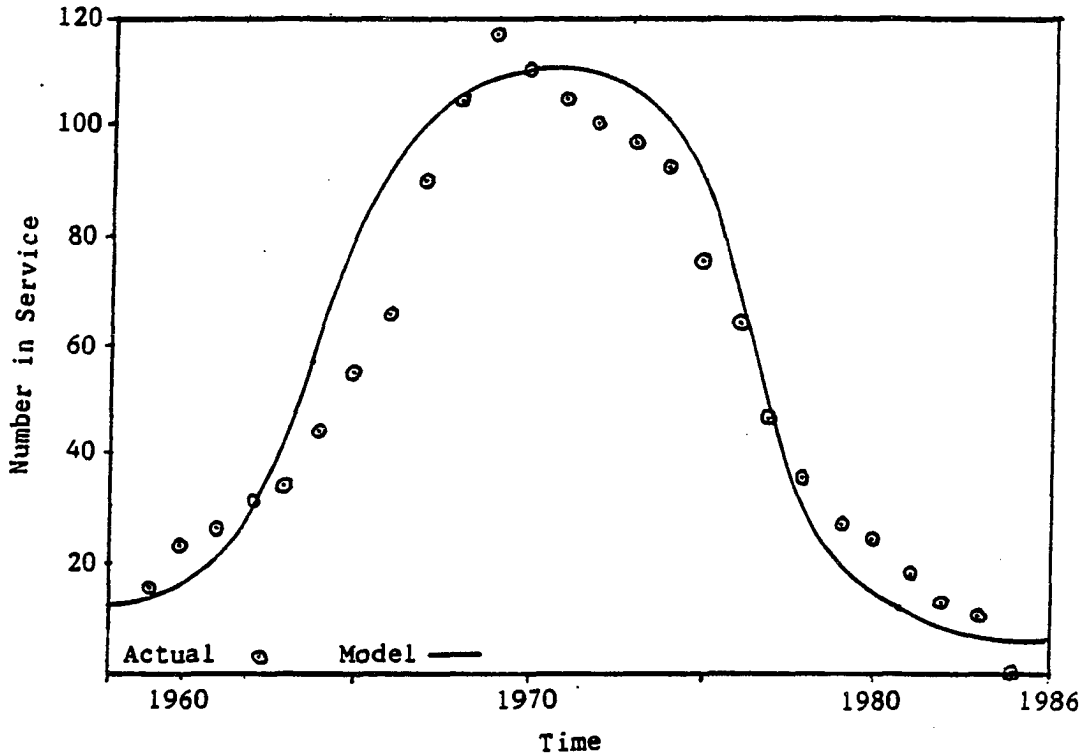


FIGURE 20. Actual and fitted life cycles for B-707s

type curve which best fits the present life cycle experience of an existing technology, it may be possible to estimate the future configuration of the life cycle, and so estimate the time over which capital should be recovered.

For this purposes tables for the values of the differential equations of the life cycle type curves are developed for different values of the time span required for the account to maximize. Using the coefficient estimates of the life cycle type curves, the ordinate values "y" for each curve are calculated for yearly increments corresponding to an unit time span. These values along with the

corresponding time units are substituted in the right hand side of the differential equation:

$$y * (1 - y) * (1 - t) * t^2,$$

where t is the yearly increment related to some value of the time span which is taken as unity. Graphs for each type life cycle rate curve are plotted on a common time scale corresponding to each value of the time span T^* required for maximum investment B^* .

To apply these curves to actual data, some additional information regarding the data is necessary. The primary data are available from the accounting department, and from the records maintained by the depreciation personnel. The additional information is to be gathered from the construction, marketing, and engineering departments. From the forecasts based on market surveys, economic demand models, and analyses of in-house billing and customer service data, it is possible for the subject matter experts to estimate the potential or the maximum investment that can be made in a particular type of technology or account.

The proportion existing investments can then be computed based on this estimate of maximum investment size. These correspond to the y 's defined earlier. By trial and error, using different time spans, these data can be fitted to the type curve graphs. Having identified a type curve, a capital recovery life can be computed from the area under the life cycle.

The above work and efforts may be very much reduced with the use of personal computers and readily available software such as LOTUS-123.

The tables can be set up within this spreadsheet program with the maximum time span as a variable which can be changed interactively. The graph representation is possible with the help of the graph subroutine such that it is possible to observe the changing graphs as the time span is changed. Using actual data in a similar manner, it is very convenient to match it to a life cycle type curve.

1. Example

To illustrate the above procedure line data of an existing electronic switching account is considered. The data are presented in Table 7; it extends from 1966 to 1990. For this example it is assumed that the information is available upto 1984, and further the maximum lines herein are to be 9000. Based on this figure the proportion end-of-year balances are calculated by dividing each year's balance by this maximum amount. Using these values in the differential rate equation, values for the account life cycle rate curve can be computed for different maximum time spans; in other words, the time axis should be scaled until the values have a close match. Comparing these values to the corresponding maximum time span type life cycle rate curves, one can find that type rate curve which closely fits the account rate curve. In this case it is the J curve at 25 years maximum time span (i.e., $T^* = 25$) (Fig. 21). Converting, the life cycle curve can be determined as shown in the Fig. 22.

TABLE 7. Line data for ESA account

| Company: PB | | Account: ESA | |
|--|------|--------------|------|
| Year :: EOY Balances (Number of Lines, '000) | | | |
| 1967 | 8 | 1979 | 2465 |
| 1968 | 12 | 1980 | 3308 |
| 1969 | 22 | 1981 | 4250 |
| 1970 | 65 | 1982 | 5173 |
| 1971 | 141 | 1983 | 6314 |
| 1972 | 310 | 1984 | 7293 |
| 1973 | 480 | 1985 | 7616 |
| 1974 | 632 | 1986 | 8175 |
| 1975 | 883 | 1987 | 8612 |
| 1976 | 1122 | 1988 | 8752 |
| 1977 | 1471 | 1989 | 8991 |
| 1978 | 1870 | 1990 | 9117 |

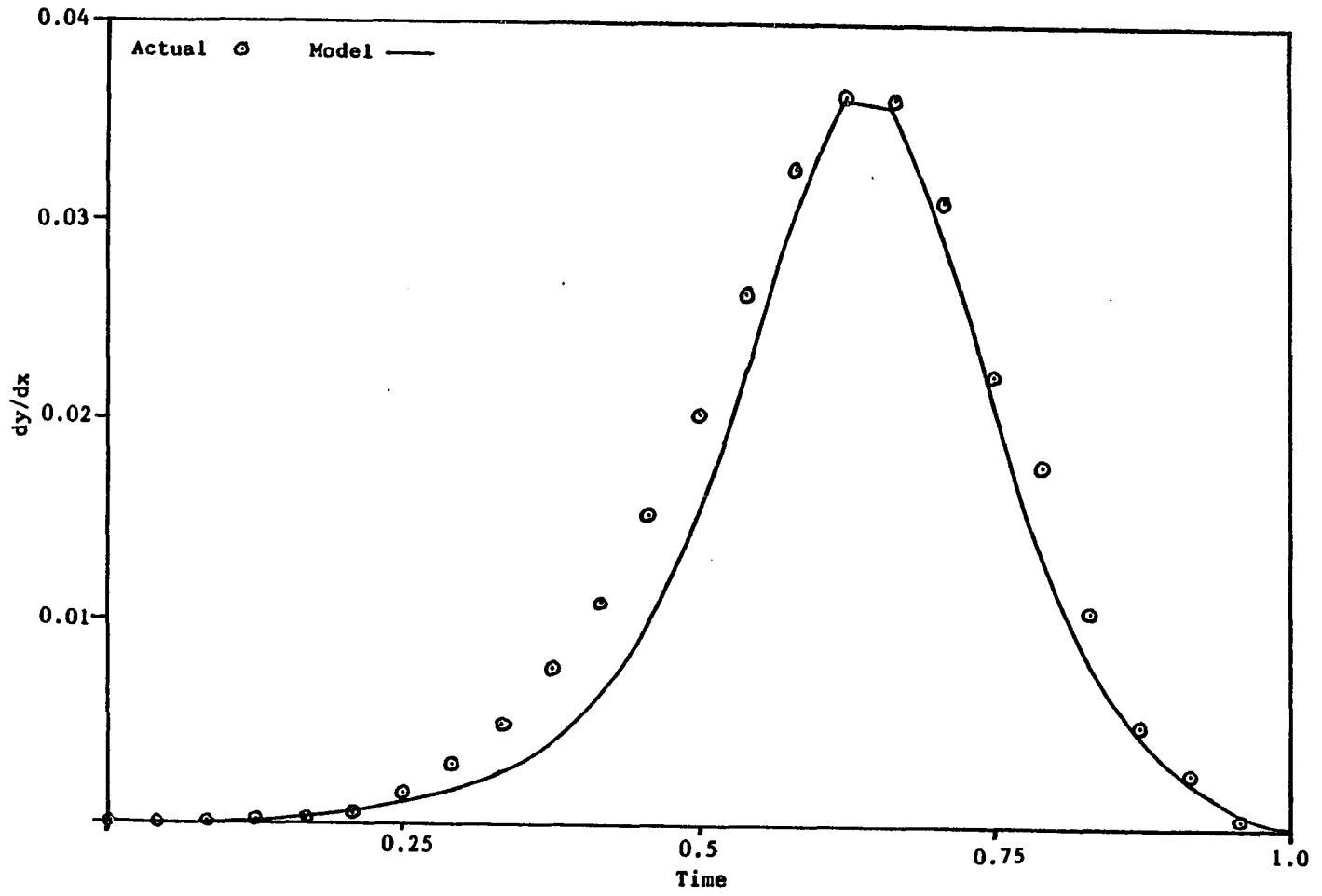


FIGURE 21. Fitting type life cycle rate curves

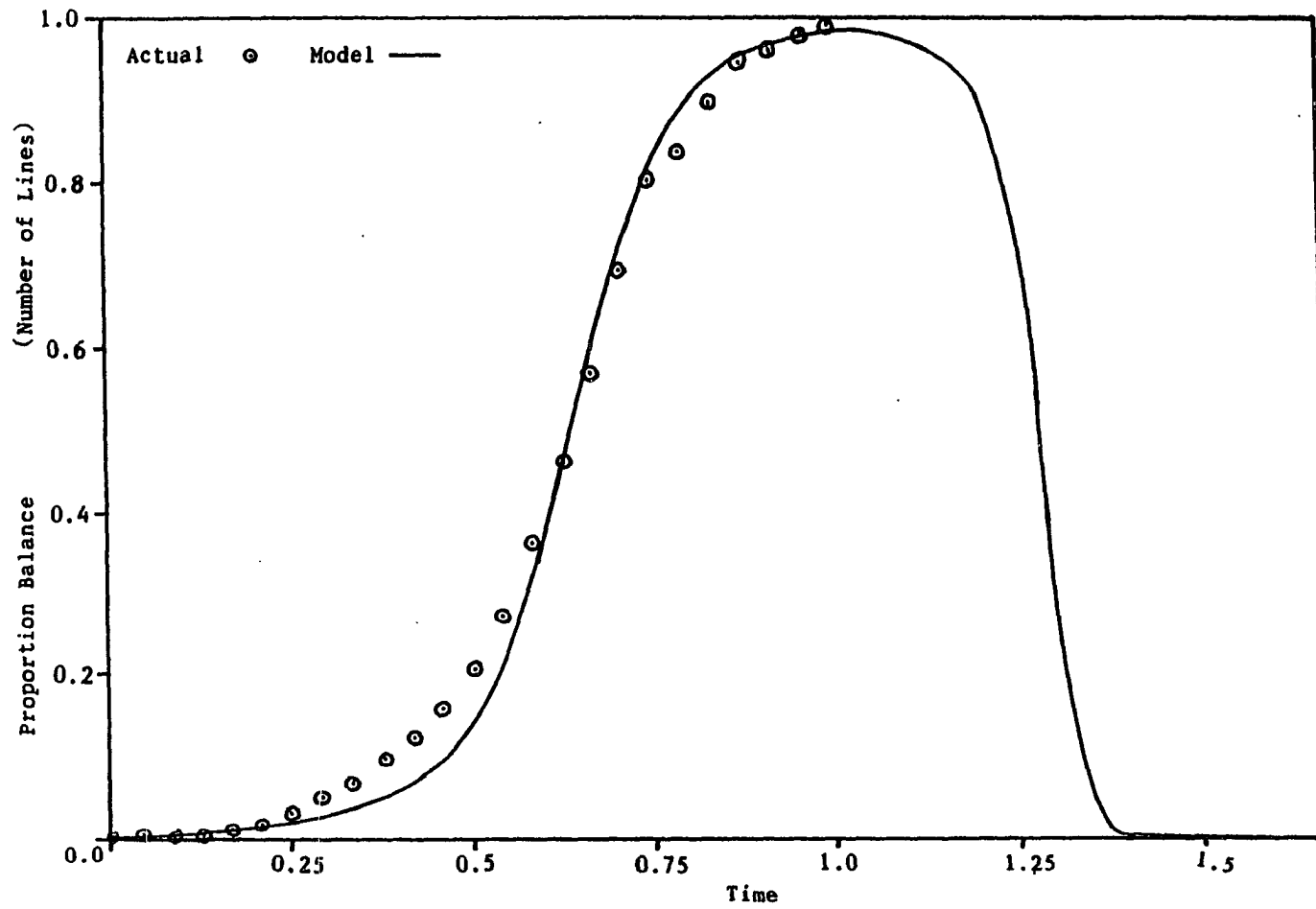


FIGURE 22. Life cycle for existing ESA account

VII. APPLYING ILC TO CAPITAL RECOVERY

Annual depreciation accrual charge, and the amount of accrued depreciation are of interest to the problems involving depreciation.

A. Life Indicators from Investment Life Cycles

1. Investment Recovery Life

By the very virtue of its measuring quantity, that is, balances, the life cycle presents the boundary beyond which an account does not survive. The area enclosed within the life cycle, therefore, corresponds to the service engendered from those balances. For different time spans T^* required for maximum investment B^* in an account the service, or the area under the curve, has been computed for each type life cycle curve. This is presented in Table 8. Thus, once the type life cycle curve has been determined the service from this curve can be directly found.

Dividing these services by the total additions that may be made in the account the investment recovery life (IRL) can be determined. The IRL is the weighted average service life of all the vintages in the account; it is equivalent to a broad group whole life. Accounting for salvage, a depreciation rate can be computed from the reciprocal of the IRL. If this rate is used from the first vintage through the complete life cycle, full recovery of the capital in that account can be achieved.

During the analysis of the actual data it was observed that the retirements in the growth section of a life cycle are significantly

TABLE 8. Service from type ILC for $G(t) = t^2$

| T* | Service from type curves | | | | | | |
|----|--------------------------|--------|--------|--------|--------|--------|--------|
| | H | C | K | O | J | T | D |
| 15 | 12.289 | 11.081 | 10.564 | 9.876 | 9.632 | 11.005 | 11.845 |
| 20 | 16.354 | 14.759 | 14.071 | 13.161 | 12.838 | 14.673 | 15.758 |
| 25 | 20.417 | 18.435 | 17.579 | 16.446 | 16.046 | 18.341 | 19.672 |
| 30 | 24.481 | 22.112 | 21.087 | 19.731 | 19.254 | 22.007 | 23.585 |
| 35 | 28.543 | 25.788 | 24.596 | 23.015 | 22.461 | 25.674 | 27.498 |
| 40 | 32.607 | 29.465 | 28.105 | 26.301 | 25.669 | 29.341 | 31.411 |
| 45 | 36.667 | 33.142 | 31.613 | 29.585 | 28.876 | 33.008 | 35.324 |
| 50 | 40.733 | 36.818 | 35.122 | 32.869 | 32.084 | 36.675 | 39.237 |
| 55 | 44.796 | 40.495 | 38.631 | 36.155 | 35.292 | 40.342 | 43.151 |
| 60 | 48.859 | 44.171 | 42.139 | 39.439 | 38.499 | 44.009 | 47.063 |
| 65 | 52.923 | 47.848 | 45.647 | 42.724 | 41.707 | 47.676 | 50.976 |
| 70 | 56.986 | 51.524 | 49.156 | 46.009 | 44.915 | 51.343 | 54.889 |
| 75 | 60.049 | 55.201 | 52.664 | 49.294 | 48.122 | 55.010 | 58.803 |

small. This is again evident from the longer lives indicated for the earlier vintages by the traditional life analysis techniques. Based on this observation it can be assumed that while no retirements occur during the growth section, the retirement rates are common in the decline section and are equal to that of the life cycle. Applying this assumption, the total additions are equal to the maximum investment balance B^* in the life cycle. As the type life cycle curves are based on normalized balances, the service indicated from these curves is then the investment recovery life.

2. Remaining Lives over the Life Cycle

From the service depicted by the type life cycle curves, and knowing the service rendered so far by the embedded plant it is possible to calculate the unrealized or future service. Johnson [26] suggests the use of information from accounting, and marketing research departments to estimate the future additions and retirements into an account. Prorating the future retirements from the retirement ratios developed from these estimates, the remaining service and so the remaining life of the embedded plant can be computed.

This approach utilizes the classical retirement rate method used by the traditional life analysis techniques. It is as reliable as the reliability of the estimated forecasts and the confidence thereon. It has been observed that the assumption of retirements being independent of age holds good in the decline section of the life cycle; moreover the retirement rates are not common to all vintages. All this constrains the application of the approach to a small area towards the tail of the life cycle.

The application was tried out on simulated data extending the number of future additions to two and three vintages. From this study it was determined that the retirement ratios of the future additions influence those of the life cycle. As such the calculated remaining life of the embedded plant is longer than actual (i.e., from the future service simulated for the embedded plant only) if there are fewer retirements from the additions, and it is shorter than the actual if more retirements are actually from the future vintages. A simple

counter example is from a simulated account consisting of vintages having identical life characteristics. The assumption of retirements being independent of age cause the embedded plant to survive right upto the end of the life cycle instead of ending at some prior time. This increases the service area, and thereby the remaining life of the embedded plant.

Johnson's approach, as pointed out earlier, has proper theoretical basis; its limitations are the quantity and quality of the data. Moreover, the subjective nature of the data makes it susceptible to criticism. Proportion balances from the type life cycle curves on the other hand are a more objective source of input. However, while the former approach requires information regarding additions and retirements, the type life cycle curves provide only the proportion balances, the resultant of the additions and retirements.

The unrealized service depicted by the selected type life cycle curve can be prorated among the survivors of the vintages constituting the embedded balance. Adding this to the corresponding realized lives of the present vintages, the average service lives for the vintages can be determined. Alternately, as total service is the sum of the realized service and the unrealized service, the weighted average remaining life of the embedded plant can be found by subtracting the weighted average realized life from the investment recovery life. The derivation of this relation is explained in the following section.

B. Accrued Depreciation

A standard practice for checking life estimation is through depreciation reserve. If the probable average service life (PASL) and net salvage of each vintage in a placement band are estimated correctly, then with the full recovery of the original cost, less net salvage, over the years the dollars of that band are in service the depreciation reserve is zero at the end of the life span of that placement band. Similarly depreciation reserve for an account should be zero at the end of the life cycle of the account.

Tsai [45] studied the depreciation reserve requirements reflecting the impact of various related characteristics in the life cycles. He investigated four life characteristics (life cycle shape, type survivor curve, vintage average service life, and range of annual additions), singly and in combination for technological change effects, for their impact on the life cycle reserve requirements. The research was based on simulated life cycles generated from assumed additions and growth rates with known life characteristics. The results are in agreement to the theoretical basis of the assumptions originating in earlier literature [48].

The assumptions regarding the additions and their growth rates add further constraints to the data generated by life cycle curves. The information normally available from the type life cycle curves is the balance ratios with respect to the maximum investment in an account. Knowledge regarding any future activities such as additions, retirements, and transfers is at best an educated estimate within the

constrains of forecasted balances from the type life cycle curves. To reduce such complications, it is convenient to develop relations with historical data, and minimal forecasted parameters.

Accrued depreciation, which unlike the depreciation reserve, does not contain retirements and salvage, and is the summation of the depreciation accruals for the preceding years. Thus by the closure of an account, accrued depreciation will amount to the depreciable base while depreciation reserve will be zero. Total accrued depreciation at any point in time is that amount which when combined with expected future annual accruals equals the depreciable base, i.e., the sum of all additions less future net salvage.

$$\sum_{1}^z D_i + \sum_{z+1}^n D_i = \sum_{1}^z A_i + \sum_{z+1}^n A_i - \sum_{1}^n A_i(s_a)$$

where

$$\begin{aligned} \sum_{1}^z D_i &= \text{accumulated depreciation at the end of year } z \\ \sum_{z+1}^n D_i &= \text{sum of future depreciation charges} \\ \sum_{1}^z A_i &= \text{total additions upto year } z \\ \sum_{z+1}^n A_i &= \text{sum of future additions} \\ \sum_{1}^n A_i(s_a) &= \text{expected net salvage} \\ s_a &= \text{average net salvage ratio.} \end{aligned}$$

Rearranging the terms,

$$\sum_1^z D_i = (1 - s_p) \sum_1^z A_i + (1 - s_f) \sum_{z+1}^n A_i - d \left\{ \begin{array}{l} \text{sum of future} \\ \text{average plant} \\ \text{balances} \end{array} \right\}$$

where $d = (1 - s_a)/\text{IRL}$ is the depreciation rate

s_p = past net salvage ratio

s_f = future net salvage ratio.

$$\left\{ \begin{array}{l} \text{sum of future} \\ \text{average plant} \\ \text{balances} \end{array} \right\} = \left\{ \begin{array}{l} \text{future service} \\ \text{from present} \\ \text{additions} \end{array} \right\} + \left\{ \begin{array}{l} \text{future service} \\ \text{from future} \\ \text{additions} \end{array} \right\}$$

Therefore we have

$$\begin{aligned} \sum_1^z D_i &= (1 - s_p) \sum_1^z A_i - d \left\{ \begin{array}{l} \text{future service} \\ \text{from present} \\ \text{additions} \end{array} \right\} \\ &\quad + (1 - s_f) \sum_{z+1}^n A_i - d \left\{ \begin{array}{l} \text{future service} \\ \text{from future} \\ \text{additions} \end{array} \right\} \end{aligned}$$

For correctly estimated depreciation rate and net salvage,

$$(1 - s_f) \sum_{z+1}^n A_i - d \left\{ \begin{array}{l} \text{future service} \\ \text{from future} \\ \text{additions} \end{array} \right\} = 0.$$

Thus

$$\sum_1^z D_i = (1 - s_p) \sum_1^z A_i - ((1 - s_a)/\text{IRL}) \left\{ \begin{array}{l} \text{future service} \\ \text{from present} \\ \text{additions} \end{array} \right\}$$

Multiplying the last term on the right hand side by $\frac{\sum_1^z A_i}{\sum_1^z A_i}$

one gets

$$\sum_1^z D_i = \sum_1^z A \left\{ (1 - s_p) - \left\{ \frac{1 - s_a}{\text{IRL}} \left\{ \begin{array}{l} \text{weighted average} \\ \text{remaining life} \\ \text{of prior additions} \end{array} \right\} \right\} \right\} \quad (5)$$

$$\sum_{1}^z D_i = d \sum_{1}^z B_i \text{ where } B_i \text{ is end of year balance.}$$

$$\frac{\sum_{1}^z B_i}{\sum_{1}^z A_i} = \frac{IRL}{(1 - s_a)} \left\{ (1 - s_p) \frac{1 - s_a}{IRL} WARM_L \right\}$$

Or

$$WAR_{eL} = (IRL / (1 - s_a)) \left\{ (1 - s_p) - ((1 - s_a) / IRL) WARM_L \right\}$$

where WAR_{eL} = weighted average realized life, and

$WARM_L$ = weighted average remaining life.

If $s_a = s_p = 0$ then,

$$WAR_{eL} = IRL - WARM_L \text{ or}$$

$$WARM_L = IRL - WAR_{eL} \quad (6)$$

This $WARM_L$ is that for the embedded plant, and it is based on known facts such as WAR_{eL} and one estimated parameter, IRL .

Substituting the value of $WARM_L$ in Eq. (5) at different points in time, one can calculate the required accumulated depreciation upto those points in time.

Eq. (5) indicates that the required accumulated depreciation for embedded plant in an account at the end of year z is dependent on:

1. the total amount of investment or additions made into the account upto year z ,
2. the past net salvage ratio correctly accounted at the end of year z ,
3. the estimated average net salvage ratio for the entire span

- of the account,
4. the weighted average remaining life of the embedded plant based on
 5. the IRL.

Table 9 and Fig. 23 illustrate the application of equations (5) and (6) to a simulated life cycle. The assumption of additions only in the growth section with zero retirements until the maximum investment is applied as the only information available is the end of year balance ratios.

C. Investment Life Cycles and Depreciation

It has been pointed out earlier that huge depreciation reserve deficits exist in some categories of property accounts. These deficits have been calculated using traditional methods of life analysis with the hindsight of more retirement experience from those accounts. Also from the life analysis results presented in Chapter IV, one can see how these deficits could have risen:

1. Initial low depreciation rates due to long lives indicated by low retirement experience.
2. Earlier vintages finally experiencing lower lives than those indicated earlier due to increased retirements.
3. Lives of newer vintages inherently smaller than those of earlier vintages.

This points out that even if life analysis, and rate determination are conducted dynamically, timely recovery of capital may not be possible.

TABLE 9. Required accumulated depreciation^a

| (1) | (2) | (3) | (4) | (5) | (6) |
|------|----------|-----------|----------|----------|----------|
| 0 | 0.000000 | 12.739300 | 0.000000 | 0.000000 | 0.000000 |
| | 0.000083 | 12.729420 | 0.000006 | 0.000006 | 0.000006 |
| | 0.002437 | 12.704570 | 0.000183 | 0.000177 | 0.000183 |
| | 0.025272 | 12.639500 | 0.002022 | 0.001839 | 0.002022 |
| | 0.110286 | 12.487280 | 0.010050 | 0.008027 | 0.010050 |
| 0.25 | 0.244390 | 12.174300 | 0.027837 | 0.017787 | 0.027837 |
| | 0.353148 | 11.656270 | 0.053541 | 0.025703 | 0.053541 |
| | 0.408820 | 10.939930 | 0.083296 | 0.029755 | 0.083296 |
| | 0.427309 | 10.061050 | 0.114398 | 0.031101 | 0.114398 |
| | 0.432397 | 9.104355 | 0.145869 | 0.031471 | 0.145869 |
| 0.5 | 0.444462 | 8.230156 | 0.178219 | 0.032349 | 0.178219 |
| | 0.479024 | 7.627647 | 0.213084 | 0.034865 | 0.213084 |
| | 0.546245 | 7.379748 | 0.252842 | 0.039757 | 0.252842 |
| | 0.645552 | 7.366374 | 0.299901 | 0.047058 | 0.299901 |
| | 0.762824 | 7.337760 | 0.355422 | 0.055521 | 0.355422 |
| 0.75 | 0.864570 | 7.091114 | 0.418349 | 0.062926 | 0.418349 |
| | 0.931774 | 6.570612 | 0.486167 | 0.067818 | 0.486167 |
| | 0.967129 | 5.832678 | 0.556558 | 0.070391 | 0.556558 |
| | 0.983154 | 4.961553 | 0.628116 | 0.071557 | 0.628116 |
| | 0.989624 | 4.018937 | 0.700145 | 0.072028 | 0.700145 |
| 1 | 0.991184 | 3.034241 | 0.772287 | 0.072142 | 0.772287 |
| | 0.987942 | 2.037512 | 0.844193 | 0.071906 | 0.844193 |
| | 0.968697 | 1.060200 | 0.914699 | 0.070505 | 0.914699 |
| | 0.832532 | 0.220264 | 0.975294 | 0.060594 | 0.975294 |
| | 0.214309 | 0.004048 | 0.990892 | 0.015598 | 0.990892 |
| 1.25 | 0.004000 | 0.000012 | 0.991183 | 0.000291 | 0.991183 |
| | 0.000012 | 0.000000 | 0.991184 | 0.000000 | 0.991184 |
| | 0.000000 | 3.1E-13 | 0.991184 | 0.000000 | 0.991184 |

^a Col 1: Standard Time ($T^* = 20$ years)

Col 2: Proportion Balance ($b_i = B_i/B^*$)

Col 3: WAR_{mL} ($= IRL - \sum_{i=1}^z b_i/b_z$ where $IRL = 13.739$)

Col 4: Required Accumulated Depreciation (from Eq. 5)

Col 5: Annual Depreciation Charge ($= b_i/IRL$, $s = .0$)

Col 6: Accumulated Depreciation ($=$ summation of Col 5 upto i).

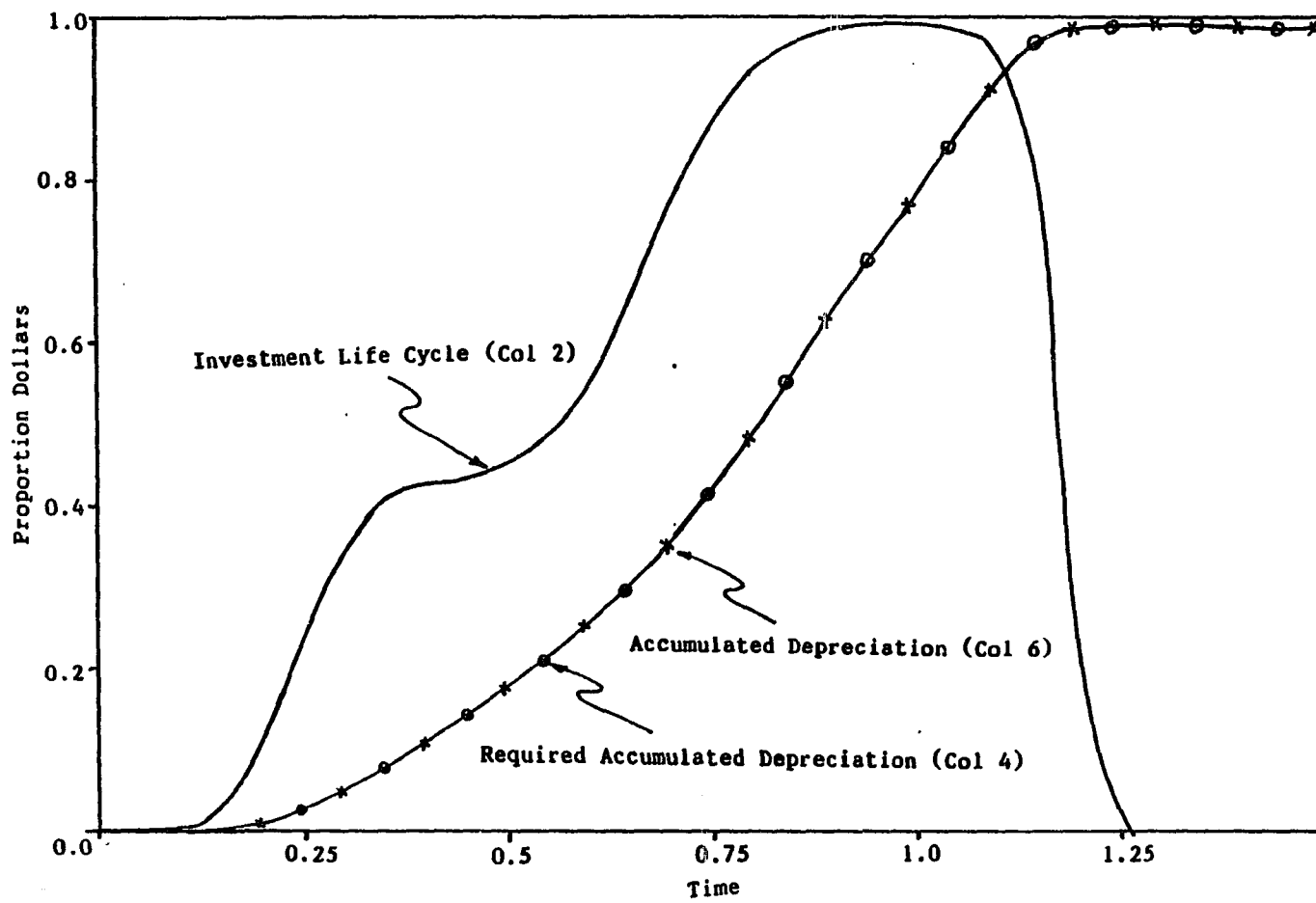


FIGURE 23. Required accumulated depreciation

Again, as whole life technique requires that the estimated life be applied to a vintage from its beginning, the changing life characteristics as retirement experience increases does cause concern when computing depreciation charges using whole life procedures.

There comes a time when one has to amortize the deficit, and/or change over to remaining life procedure. Both these alternatives require the estimation of the future service expected from the investment or account. Life cycle analysis can provide a helping hand in that area.

At least three scenarios can be drawn to apply investment life cycles to the calculation of depreciation, and the process of capital recovery.

1. An IRL be estimated at the very beginning of the account, and applied from the start for estimating proper depreciation rates. The estimated investment life cycle should be adhered to within practical limits, and any deviation from this life cycle happening in reality may be separately accounted as the situation arose.
2. At any point in the experience of an account, an estimated life cycle will set the limits to service expected from the account. At this time, combining the future expected service with the realized service of each existing vintage, one can calculate a corrected service life for each embedded placement, and compute depreciation using whole life procedure to ensure complete recovery of capital by the end

of the life cycle.

3. A remaining life for embedded plant can be determined by estimating a life cycle at that point in the experience of the account. Future depreciation charges can be computed using remaining life procedure.

A convenient way of checking proper depreciation allocation is through accumulated depreciation. Based on an IRL, estimated by any means, it is possible to determine the required accumulated depreciation for the embedded investment. Should this figure not tally with actual accumulated depreciation, corrective action is necessary.

VIII. SUMMARY AND RECOMMENDATIONS

Through life analysis of actual data it has been observed that service life indicators suddenly reduce when high retirement rates set in. Technological advancements, obsolescence, and competition are some of the reasons that cause the increase in the retirement rates. When these reasons are not properly incorporated into the life estimation process, timely recovery of capital is not possible; this is evident from the huge depreciation reserve deficit experienced by some telephone companies in the eighties.

Objective analysis of these causes of retirement is a formidable task. However, they can be indirectly included by augmenting the age dependent traditional life analysis process with some type of time related procedure. The product life cycle is one such concept that can be used profitably.

It has been shown in this study that the product life cycle concept, commonly used in marketing management, can be adopted to be applied in the process of capital recovery. From the concept of investment life cycle developed to suit the depreciation parlance an investment recovery life (IRL) has been defined. Investment recovery life from life cycle analysis is the weighted average of service lives of all the vintages in an account developed from traditional life analysis. Based on this relation shorter service lives can be argued for the initial vintages even though traditional life analysis techniques may portray longer lives at that time.

From the argument that the product life cycle concept has its roots in the diffusion theory, a generalized model for the investment life cycle has been developed. Although each product has its unique life cycle configuration, the model developed here can be modified to describe life cycles of different investments. Based on this model, and from its application to several data sets, type life cycle curves have been developed (a concept similar to Iowa type survivor curves). A routine for applying these type life cycle curves to existing investment is proposed and demonstrated. An IRL can be derived from this application. When a depreciation rate computed from this IRL is applied to an account from its beginning complete recovery of the capital invested in that account can be accomplished.

The investment life cycle approach can be seen to be a much wider application of the broad group whole life system of depreciation. The width of the broad group extends across the whole span of the account. Although this tends to hide many details of vintage analysis, the life cycle approach gives the first approximation and so sets the limits within which timely recovery of capital is possible. With the projected life cycle and IRL derived from type life cycle curves, the remaining life and required accumulated depreciation of embedded plant can be determined. The formulae for remaining life, and accumulated depreciation have been derived, and their application has been illustrated.

This study has presented product life cycle as a versatile conceptual tool, although it may appear to be superficial and/or

simplistic. The difficulty lies in the successful presentation of a model common for all products or investments. The model presented herein can be modified to suit different situations. The type curves generated from this model represent a range of property affected by a certain degree of technological obsolescence, and competition. Different types of properties under varying conditions of obsolescence, and competition need to be analyzed to get a complete picture of the life cycle. Additional information from the analyses of such data would weave the intricacy of additions and retirements internal to the outer coat of investment life cycle.

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XI. APPENDIX. SAMPLE RESULTS OF MORTALITY ANALYSIS OVER LIFE CYCLES

TABLE 10. Mortality analysis A

| Vintage | Iowa type curve | ASL (years) |
|---------|-----------------|-----------------|
| | Company: CI | |
| | Account: XBAR | |
| 1940 | R 2 | 97.7 |
| 1941 | S 0.5 | 88.8 |
| 1942 | R 3 | 61.3 |
| 1943 | -- | -- |
| 1944 | -- | -- |
| 1945 | S 6 | 38.8 |
| 1946 | S 6 | 35.0 |
| 1947 | S 6 | 34.0 |
| 1948 | S 5 | 33.9 |
| 1949 | R 4 | 31.3 |
| 1950 | R 2.5 | 40.5 |
| 1951 | L 5 | 29.1 |
| 1952 | R 1.5 | 33.8 |
| 1953 | S 5 | 28.0 |
| 1954 | L 4 | 28.6 |
| 1955 | L 3 | 35.1 |
| 1956 | L 0 | 44.0 |
| 1957 | L 4 | 25.3 |
| 1958 | L 4 | 24.1 |
| 1959 | R 3 | 25.1 |
| 1960 | S 5 | 20.5 |
| 1961 | R 3 | 23.6 |
| 1962 | R 2.5 | 23.9 |
| 1963 | S 3 | 21.6 |
| 1964 | S 2 | 22.6 |
| 1965 | S 3 | 18.2 |
| 1966 | S 1.5 | 24.7 |
| 1967 | L 3 | 15.6 |
| 1968 | L 4 | 13.7 |
| 1969 | L 4 | 13.9 |
| 1970 | L 3 | 14.1 |
| 1971 | S 1.5 | 16.5 |
| 1972 | S 1.5 | 13.6 |
| 1973 | S 1 | 13.0 |
| 1974 | L 1.5 | 11.3 |
| 1975 | S 1 | 10.8 |
| 1976 | L 0.5 | 8.8 |
| 1977 | R 0.5 | 14.0 |
| 1978 | ID | ID ^a |
| 1979 | R 0.5 | 9.0 |

^a Insufficient Data.

TABLE 11. Mortality analysis B

| Vintage | Iowa type curve | ASL (years) |
|---------|-----------------|-------------|
| | Company: CI | |
| | Account: SXS | |
| 1937 | | |
| 1938 | R 1.5 | 81.8 |
| 1939 | L 0 | 34.9 |
| 1940 | R 0.5 | 37.9 |
| 1941 | L 0 | 32.7 |
| 1942 | L 0.5 | 36.2 |
| 1943 | S 6 | 38.0 |
| 1944 | R 1 | 40.8 |
| 1945 | R 0.5 | 39.3 |
| 1946 | S 6 | 30.1 |
| 1947 | R 2 | 26.5 |
| 1948 | R 5 | 29.5 |
| 1949 | R 1.5 | 27.0 |
| 1950 | L 1 | 19.1 |
| 1951 | R 3 | 26.7 |
| 1952 | R 3 | 25.2 |
| 1953 | R 2 | 20.8 |
| 1954 | R 5 | 24.6 |
| 1955 | S 4 | 21.3 |
| 1956 | L 4 | 22.1 |
| 1957 | L 2 | 19.9 |
| 1958 | L 5 | 17.7 |
| 1959 | L 2 | 18.8 |
| 1960 | S 2 | 17.2 |
| 1961 | L 1.5 | 16.5 |
| 1962 | R 2.5 | 17.2 |
| 1963 | S 2 | 14.0 |
| 1964 | S 1.5 | 14.0 |
| 1965 | L 1.5 | 15.1 |
| 1966 | R 2.5 | 18.9 |
| 1967 | L 1 | 12.0 |
| 1968 | S 2 | 11.8 |
| 1969 | L 5 | 12.4 |
| 1970 | S 0 | 8.6 |
| 1971 | L 0.5 | 9.6 |
| 1972 | R 3 | 9.0 |
| 1973 | L 4 | 9.1 |
| 1974 | L 1.5 | 9.7 |
| 1975 | S 6 | 6.0 |
| 1976 | L 2 | 6.7 |

TABLE 11. Continued

| Vintage | Iowa type curve | ASL (years) |
|---------|-----------------------------|-------------|
| | Company: CI Account: SXS | |
| 1977 | L 3 | 5.5 |
| 1978 | L 4 | 3.6 |
| 1979 | L 0.5 | 4.2 |
| 1980 | L 1 | 3.9 |

TABLE 12. Mortality analysis C

| Vintage | Iowa type curve | ASL (years) |
|---------|-----------------|-------------|
| | Company: ID | |
| | Account: XBAR | |
| 1937 | S 6 | 42.2 |
| 1938 | L 5 | 39.8 |
| 1939 | S 3 | 60.2 |
| 1940 | R 0.5 | 74.7 |
| 1941 | R 1.5 | 56.0 |
| 1942 | R 3 | 41.6 |
| 1943 | S 6 | 38.1 |
| 1944 | R 1 | 38.6 |
| 1945 | R 0.5 | 49.4 |
| 1946 | R 5 | 34.6 |
| 1947 | R 4 | 33.2 |
| 1948 | R 3 | 31.5 |
| 1949 | R 4 | 32.8 |
| 1950 | R 3 | 31.3 |
| 1951 | R 4 | 27.9 |
| 1952 | F 4 | 30.3 |
| 1953 | R 2.5 | 32.6 |
| 1954 | R 4 | 26.1 |
| 1955 | R 4 | 27.2 |
| 1956 | R 5 | 25.0 |
| 1957 | L 5 | 25.1 |
| 1958 | L 3 | 26.4 |
| 1959 | S 2 | 29.8 |
| 1960 | L 3 | 25.8 |
| 1961 | L 3 | 21.7 |
| 1962 | L 4 | 18.7 |
| 1963 | R 4 | 18.7 |
| 1964 | L 3 | 19.6 |
| 1965 | R 3 | 17.2 |
| 1966 | R 3 | 17.2 |
| 1967 | L 3 | 16.8 |
| 1968 | R 3 | 14.6 |
| 1969 | R 3 | 13.5 |
| 1970 | S 1.5 | 12.8 |
| 1971 | S 2 | 11.2 |
| 1972 | L 2 | 11.8 |
| 1973 | R 3 | 10.4 |
| 1974 | S 2 | 8.9 |
| 1975 | L 1.5 | 10.6 |
| 1976 | S 1 | 8 |

TABLE 12. Continued

| Vintage | Iowa type curve | ASL (years) |
|---------|-----------------|-------------|
| | Company: ID | |
| | Account: XBAR | |
| 1977 | R 2 | 8.2 |
| 1978 | L 1 | 8 |
| 1979 | L 1.5 | 7 |
| 1980 | R 2.5 | 4.5 |
| 1981 | L 2 | 4.1 |

TABLE 13. Mortality analysis D

| Vintage | Iowa type curve | ASL (years) |
|---------|------------------|-------------|
| | Company: IB | |
| | Account: SXS CDO | |
| 1937 | R 2 | 25.6 |
| 1938 | S 1 | 26.1 |
| 1939 | S 1 | 24.9 |
| 1940 | R 3 | 29.8 |
| 1941 | R 1 | 29.0 |
| 1942 | S 0 | 29.0 |
| 1943 | R 1 | 26.6 |
| 1944 | R 3 | 28.4 |
| 1945 | R 1.5 | 25.9 |
| 1946 | S 0 | 24.1 |
| 1947 | R 1 | 23.5 |
| 1948 | R 2 | 25.3 |
| 1949 | R 1 | 26.6 |
| 1950 | R 1 | 26.4 |
| 1951 | S -.5 | 18.8 |
| 1952 | R 1.5 | 24.9 |
| 1953 | R 1 | 23.4 |
| 1954 | R 2 | 25.2 |
| 1955 | R 2 | 22.1 |
| 1956 | R 1.5 | 23.7 |
| 1957 | R 1.5 | 19.1 |
| 1958 | R 1.5 | 25.0 |
| 1959 | R 2 | 20.7 |
| 1960 | L 1.5 | 25.8 |
| 1961 | L 1.5 | 20.8 |
| 1962 | R 1 | 17.4 |
| 1963 | L 1.5 | 18.3 |
| 1964 | L 2 | 14.8 |
| 1965 | L 2 | 14.5 |
| 1966 | L 1.5 | 17.3 |
| 1967 | L 1 | 16.3 |
| 1968 | L 3 | 11.6 |
| 1969 | L 0 | 13.1 |
| 1970 | L 0 | 12.7 |
| 1971 | L 0 | 9.9 |
| 1972 | L 0 | 7.7 |
| 1973 | L 1.5 | 10.2 |
| 1974 | L 0 | 7.7 |
| 1975 | L 1 | 7.2 |
| 1976 | L 0 | 6.9 |

TABLE 13. Continued

| Vintage | Iowa type curve | ASL (years) |
|---------|------------------|-------------|
| | Company: IB | |
| | Account: SXS CDO | |
| 1977 | R 0.5 | 7.2 |
| 1978 | R 0.5 | 8.9 |
| 1979 | S -.5 | 9.4 |
| 1980 | R 0.5 | 12.7 |

TABLE 14. Mortality analysis E

| Vintage | Iowa type curve | ASL (years) |
|---------|-------------------------------|-------------|
| | Company IB Account: LARSXS | |
| 1931 | S 3 | 45.3 |
| 1932 | S 1.5 | 60.3 |
| 1933 | S 6 | 42.9 |
| 1934 | S 5 | 48.3 |
| 1935 | S 6 | 39.5 |
| 1936 | S 0 | 78.4 |
| 1937 | R 5 | 37.7 |
| 1938 | L 5 | 37.8 |
| 1939 | L 2 | 80.8 |
| 1940 | S 6 | 34.4 |
| 1941 | S 1.5 | 35.1 |
| 1942 | R 4 | 32.0 |
| 1943 | L 1 | 34.9 |
| 1944 | R 4 | 30.9 |
| 1945 | L 5 | 30.6 |
| 1946 | R 4 | 27.9 |
| 1947 | R 3 | 26.4 |
| 1948 | R 4 | 25.1 |
| 1949 | R 2.5 | 31.1 |
| 1950 | R 2 | 29.8 |
| 1951 | R 4 | 21.2 |
| 1952 | R 2.5 | 20.6 |
| 1953 | R 3 | 20.3 |
| 1954 | R 3 | 19.8 |
| 1955 | R 3 | 19.9 |
| 1956 | R 2 | 17.4 |
| 1957 | R 3 | 16.7 |
| 1958 | R 1.5 | 16.4 |
| 1959 | R 1.5 | 14.3 |
| 1960 | R 3 | 13.9 |
| 1961 | L 3 | 14.5 |
| 1962 | R 4 | 13.4 |
| 1963 | S 1.5 | 12.7 |
| 1964 | R 2.5 | 10.6 |
| 1965 | R 3 | 10.1 |
| 1966 | L 3 | 9.3 |
| 1967 | S 2 | 8.2 |
| 1968 | S 1.5 | 8.4 |
| 1969 | S 1.5 | 6.4 |
| 1970 | L 1.5 | 6.2 |
| 1971 | L 1.5 | 4.9 |

TABLE 14. Continued

| Vintage | Iowa type curve | ASL (years) |
|---------|-----------------|-------------|
| | Company: IB | |
| | Account: LARSXS | |
| 1972 | L 1 | 4.8 |
| 1973 | R 1 | 3.5 |
| 1974 | L 0 | 3.7 |
| 1975 | L 0 | 4.5 |
| 1976 | L 1 | 3.8 |
| 1977 | L 0 | 3.5 |
| 1978 | S 0 | 2.9 |
| 1979 | S 4 | 1.2 |
| 1980 | S -.5 | 1.2 |